Formulaire de Mathématiques

Page

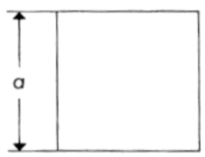
1. Généralités

1.1 Constantes mathématiques et physico-chimiques

π	π = 3,14159 26535 89793 23846 26433 83279 50288 41971 69399 37510 58209 74944 59230 78164 06286 20899 86280 34825 34211 70679 ...
e	e = 2,71828 18284 59045 23536 02874 71352 66249 77572 47093 69995 95749 66967 62772 40766 30353 54759 45713 82178 52516 64274 ...
Nombre d'Or	Φ = 1,61803 39887 49894 84820 45868 34365 63811 77203 09179 80576 28621 35448 62270 52604 62818 90244 97072 07204 18939 11374 ...
Constante d'Euler	γ = 0,57721 56649 01532 86060 65120 90082 40243 10421 59335 93992 35988 05767 23488 48677 26777 66467 09369 47063 29174 67495 ...

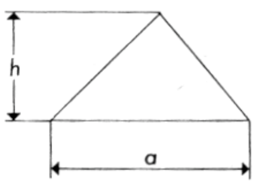
Vitesse de la lumière dans le vide	c	299792458 m.s^{-1} (*valeur exacte*)
Constante de la Gravitation Universelle	$\mathcal{G}$	6,67259.10^{-11} m^3.kg^{-1}.s^{-2}
Constante de Planck	h	6,6260755.10^{-34} J.s
Constante de Planck réduite	$\hbar$	1,05457266.10^{-34} J.s
Charge élémentaire	e	1,60217733.10^{-19} C
Masse de l'électron au repos	m_e	9,1093897.10^{-31} kg
Masse du proton au repos	m_p	1,6726231.10^{-27} kg
Masse du neutron au repos	m_n	1,6749286.10^{-27} kg
Masse du muon au repos	m_μ	1,8835327.10^{-28} kg
Unité de masse atomique	m_u	1,6605402.10^{-27} kg
Nombre d'Avogadro	$\mathcal{N}$	6,0221367.10^{23}
Constante de Boltzmann	k_B	1,380658.10^{-23} J.K^{-1}
Perméabilité magnétique du vide	μ_0	1,25663706143592.10^{-6} N.A^{-2}
Permittivité du vide	ε_0	8,854187817.10^{-12} F.m^{-1}
Constante de Stefan-Boltzmann	σ	5,67051.10^{-8} W.m^{-2}.K^{-4}
Constante de structure fine	α	0,00729735308
Constante de Rydberg	R_∞	10973731,534 m^{-1}
Rayon classique de l'électron	r_e	2,81794092.10^{-15} m
Constante des gaz parfaits	R	8,31451 J.mol^{-1}.K^{-1}

1.2 Aires remarquables



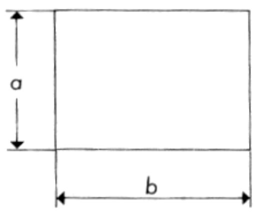
Carré

$S = a^2$



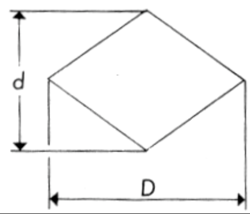
Triangle

$S = \frac{1}{2} a \times h$



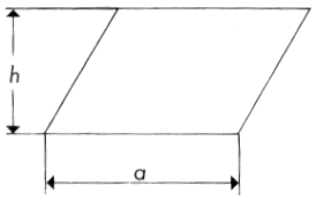
Rectangle

$S = a \times b$



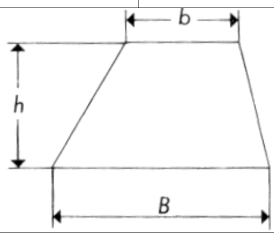
Losange

$S = \frac{1}{2} d \times D$



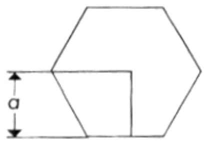
Parallélogramme

$S = a \times h$



Trapèze

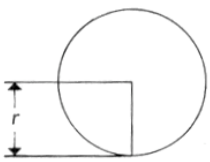
$S = \frac{b+B}{2} \times h$



Polygone régulier à *n* côtés

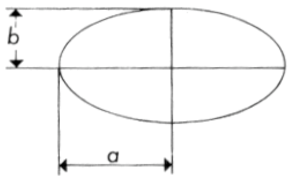
$\left(périmètre : p = 2n \times a \times \tan \frac{\pi}{n} \right)$

$S = p \times \frac{a}{2}$



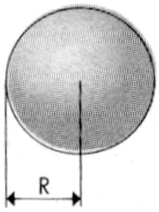
Disque

$S = \pi \times r^2$



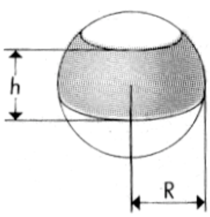
Ellipse

$S = \pi \times a \times b$



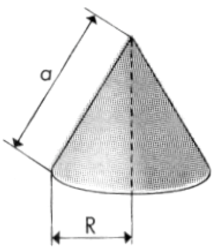
Sphère

$S = 4\pi \times R^2$



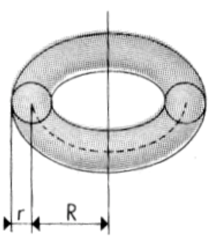
Zone sphérique

$S = 2\pi \times R \times h$



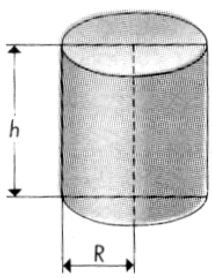
Cône

$S = \pi \times R \times a$



Tore

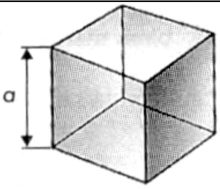
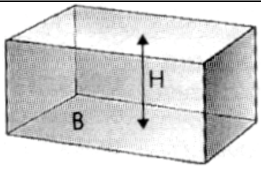
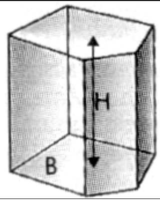
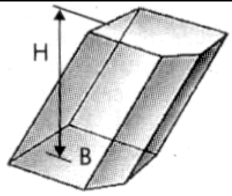
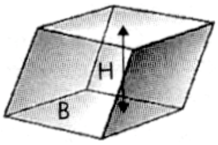
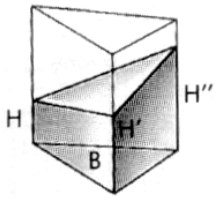
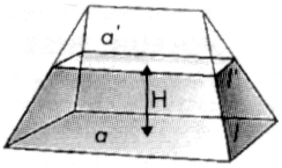
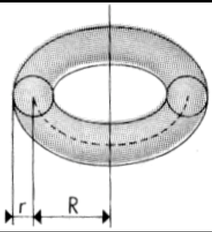
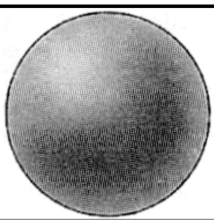
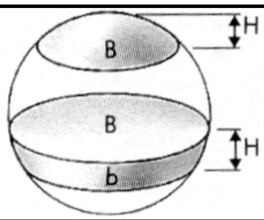
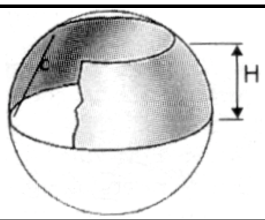
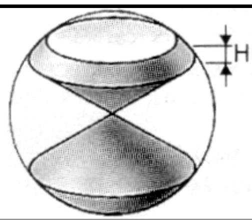
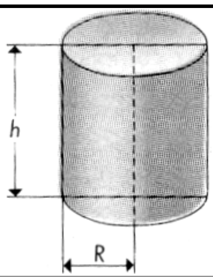
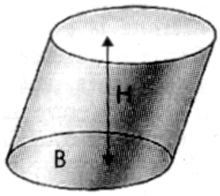
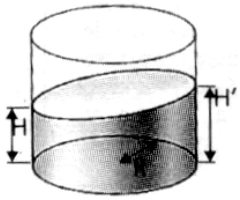
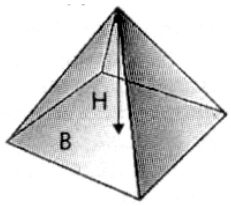
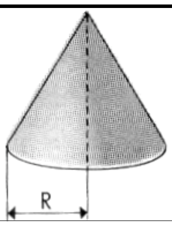
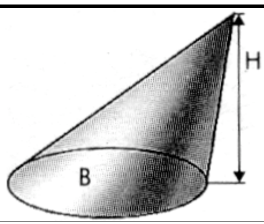
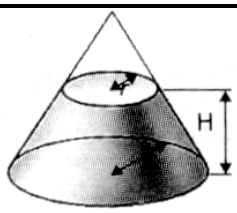
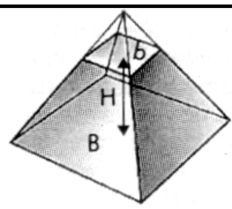
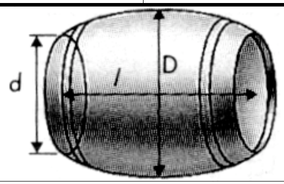
$S = 4\pi^2 \times R \times r$



Cylindre

$S = 2\pi \times R \times h$

1.3 Volumes remarquables

			
Cube : $V = a^3$	Parallélépipède : $V = B \times H$	Prisme droit : $V = B \times H$	Prisme oblique : $V = B \times H$
			
Rhomboèdre $V = B \times H$	Prisme tronqué $V = B \times \frac{H + H' + H''}{3}$	Tas de sable $V = \frac{H}{6}\left[l(2a + a') + l'(a + 2a')\right]$	Tore $V = 2\pi^2 \times r^2 \times R$
			
Sphère $V = \frac{4}{3}\pi \times R^3$	Segment sphérique $V = \frac{1}{6}\pi \times H^3 + \frac{b+B}{2} \times H$	Anneau sphérique $V = \frac{1}{6}\pi \times c^2 \times H$	Secteur sphérique $V = \frac{2}{3}\pi \times R^2 \times H$
			
Cylindre circulaire $V = \pi \times R^2 \times H$	Cylindre oblique $V = B \times H$	Cylindre tronqué $V = \pi \times R^2 \times \frac{H + H'}{2}$	Pyramide $V = \frac{1}{3} B \times H$
			
Cône circulaire $V = \frac{1}{3}\pi \times R^2 \times H$	Cône oblique $V = \frac{1}{3} B \times H$	Cône tronqué $V = \pi \times \frac{H}{3}\left(R^2 + r^2 + R.r\right)$	Pyramide tronquée $V = \frac{H}{3}\left(B + b + \sqrt{Bb}\right)$
			
Tonneau : $V = \pi \times l \times \left(\frac{D}{3} + \frac{d}{6}\right)^2$			

2. Algèbre

2.1. Identités remarquables

$x^2 - y^2$	$(x-y)(x+y)$
$x^3 - y^3$	$(x-y)(x^2+xy+y^2)$
$x^4 - y^4$	$(x-y)(x^3+x^2y+xy^2+y^3)$
$x^n - y^n$	$(x-y)(x^{n-1}+x^{n-2}y+...+x.y^{n-2}+y^{n-1})$
$x^2 - y^2$	$(x-y)(x+y)$
$x^4 - y^4$	$(x+y)(x^3-x^2y+x.y^2-y^3)$
$x^n - y^n$	$(x+y)(x^{n-1}-x^{n-2}y+...+xy^{n-2}-y^{n-1})$ pour tout n pair
$x^2 + y^2$	$(x+iy)(x-iy)$
$x^3 + y^3$	$(x+y)(x^2-xy+y^2)$
$x^n + y^n$	$(x+y)(x^{n-1}-x^{n-2}y+...-xy^{n-2}+y^{n-1})$ pour tout n impair

2.2. Sommes usuelles

$1+2+3+...+n$	$\frac{n(n+1)}{2}$
$1^2+2^2+3^2+...+n^2$	$\frac{n(n+1)(2n+1)}{6}$
$1^3+2^3+3^3+...+n^3$	$\frac{n^2(n+1)^2}{2}$
$1^4+2^4+3^4+...+n^4$	$\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
$1+3+5+...+(2n-1)$	n^2
$2+4+6+...+(2n)$	$n(n+1)$
$1^2+3^2+5^2+...+(2n-1)^2$	$\frac{n(2n-1)(2n+1)}{3}$
$2^2+4^2+6^2+...+(2n)^2$	$\frac{2n(n+1)(2n+1)}{3}$
$1^3+3^3+5^3+...+(2n-1)^3$	$n^2(2n^2-1)$
$2^3+4^3+6^3+...+(2n)^3$	$2n^2(n+1)^2$
$1.2+2.3+3.4+...+n(n-1)$	$\frac{n(n-1)(n+1)}{3}$
$1.2.3+2.3.4+...+n(n-1)(n-2)$	$\frac{n(n-2)(n-1)(n+1)}{4}$

2.3. Binôme de Newton et triangle de Pascal

$$(x+y)^n = x^n + C_n^1 x^{n-1} y + C_n^2 x^{n-2} y^2 + \dots + C_n^{n-2} x^2 y^{n-2} + C_n^{n-1} x.y^{n-1} + y^n$$

1																
1	1															
1	2	1														
1	3	3	1													
1	4	6	4	1												
1	**5**	**10**	**10**	**5**	**1**											
1	6	15	20	15	6	1										
1	7	21	35	35	21	7	1									
1	**8**	**28**	**56**	**70**	**56**	**28**	**8**	**1**								
1	9	36	84	126	126	84	36	9	1							
1	10	45	120	210	252	210	120	45	10	1						
1	11	55	165	330	462	462	330	165	55	11	1					
1	12	66	220	495	792	924	792	495	220	66	12	1				
1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1			
1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1		
1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1	
1	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	560	120	16	1

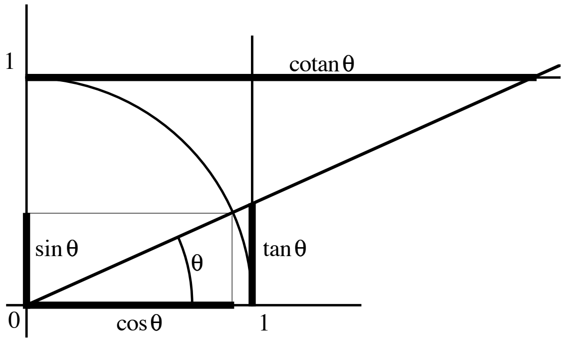
Par exemple (à l'aide des lignes en gras dans le triangle de Pascal) :

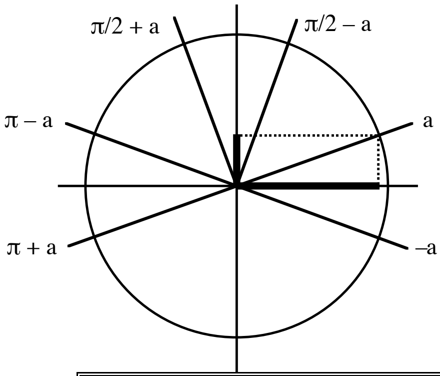
$$(x+y)^5 = x^5 + 5x^4 y + 10x^3 y^2 + 10x^2 y^3 + 5xy^4 + y^5$$

$$(x-y)^8 = x^8 - 8x^7 y + 28x^6 y^2 - 56x^5 y^3 + 70x^4 y^4 - 56x^3 y^5 + 28x^2 y^6 - 8xy^7 + y^8$$

2.4. Trigonométrie circulaire

Formule de Moivre	Formules d'Euler
$(\cos x + i \sin x)^n = \cos nx + i \sin nx$	$\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$
	$\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$





$x =$	$-a$	$\frac{\pi}{2} - a$	$\frac{\pi}{2} + a$	$\pi - a$	$\pi + a$
$\cos x =$	$\cos a$	$\sin a$	$-\sin a$	$-\cos a$	$-\cos a$
$\sin x =$	$-\sin a$	$\cos a$	$\cos a$	$\sin a$	$-\sin a$
$\tan x =$	$-\tan a$	$\text{cotan}\, a$	$-\text{cotan}\, a$	$-\tan a$	$\tan a$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$	$\cos a \cos b = \frac{1}{2}(\cos(a+b) + \cos(a-b))$	$\cos p + \cos q = 2\cos\frac{p+q}{2}.\cos\frac{p-q}{2}$
$\sin(a+b) = \sin a \cos b + \cos a \sin b$	$\sin a \sin b = \frac{1}{2}(\cos(a-b) - \cos(a+b))$	$\cos p - \cos q = -2\sin\frac{p+q}{2}.\sin\frac{p-q}{2}$
$\cos(a-b) = \cos a \cos b + \sin a \sin b$	$\cos a \sin b = \frac{1}{2}(\sin(a+b) - \sin(a-b))$	$\sin p + \sin q = 2\sin\frac{p+q}{2}.\cos\frac{p-q}{2}$
$\sin(a-b) = \sin a \cos b - \cos a \sin b$	$\sin a \cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))$	$\sin p - \sin q = 2\cos\frac{p+q}{2}.\sin\frac{p-q}{2}$

$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$	$\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$	$\tan 2a = \dfrac{2\tan a}{1 - \tan^2 a}$	$\tan 3a = \dfrac{3\tan a - \tan^3 a}{1 - 3\tan^2 a}$

$\cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$	$\cos 3a = 4\cos^3 a - 3\cos a$	$\cos^2 a = \dfrac{1 + \cos 2a}{2}$	$\cos^3 a = \dfrac{3\cos a + \cos 3a}{4}$
$\sin 2a = 2\sin a \cos a$	$\sin 3a = 3\sin a - 4\sin^3 a$	$\sin^2 a = \dfrac{1 - \cos 2a}{2}$	$\sin^3 a = \dfrac{3\sin a - \sin 3a}{4}$

2.5. Trigonométrie hyperbolique

$\text{ch}x = \dfrac{e^x + e^{-x}}{2}$	$\text{Argch}x = \ln\left(x + \sqrt{x^2 - 1}\right)$	$e^a + e^b = 2.e^{(a+b)/2}.\text{ch}\dfrac{a-b}{2}$
$\text{sh}x = \dfrac{e^x - e^{-x}}{2}$	$\text{Argsh}x = \ln\left(x + \sqrt{x^2 + 1}\right)$	$e^a - e^b = 2.e^{(a+b)/2}.\text{sh}\dfrac{a-b}{2}$
$\text{th}x = \dfrac{\text{sh}x}{\text{ch}x} = \dfrac{e^{2x} - 1}{e^{2x} + 1}$	$\text{Argth}x = \dfrac{1}{2}\ln\dfrac{1+x}{1-x}$	$\dfrac{e^a - e^b}{e^a + e^b} = \text{th}\dfrac{a-b}{2}$

2.6. Équations algébriques de degré 2

Résolution de $ax^2+bx+c=0$	• discriminant : $\Delta = b^2-4ac$ • $\begin{cases} \text{si } \Delta>0 \;:\; x_1=\dfrac{-b-\sqrt{\Delta}}{2a} \text{ et } x_2=\dfrac{-b+\sqrt{\Delta}}{2a} \\ \text{si } \Delta=0 \;:\; x_1=x_2=\dfrac{-b}{2a} \\ \text{si } \Delta<0 \;:\; x_1=\dfrac{-b-i\sqrt{-\Delta}}{2a} \text{ et } x_2=\dfrac{-b+i\sqrt{-\Delta}}{2a} \end{cases}$
Factorisation	$ax^2+bx+c=a(x-x_1)(x-x_2)$ et $\begin{cases} x_1+x_2=-\dfrac{b}{a} \\ x_1x_2=\dfrac{c}{a} \end{cases}$
Réciproque	si $\begin{cases} x+y=S \\ x.y=P \end{cases}$ alors x et y sont solutions de l'équation : $x^2-S.x+P=0$

2.7. Équations algébriques de degré 3

① Réduction de x^3+ax^2+bx+c	on pose $\begin{cases} x=y-\dfrac{a}{3} \\ p=\dfrac{b}{3}-\dfrac{a^2}{9} \\ q=\dfrac{a^3}{27}-\dfrac{ab}{6}+\dfrac{c}{2} \end{cases}$ et l'expression prend la forme $\boxed{y^3+3py+2q}$
② Résolution de $y^3+3py+2q=0$	• discriminant : $R=p^3+q^2$ • si R = 0 : $y_1=y_2=\dfrac{-q}{p}$ et $y_3=\dfrac{2q}{p}$ • si R > 0 : $\begin{cases} y_1=u+v \\ y_2=j.u+j^2v \\ y_3=j^2u+j.v \end{cases}$ avec $\begin{cases} u=\sqrt[3]{-q+\sqrt{R}} \\ v=\sqrt[3]{-q-\sqrt{R}} \end{cases}$ (formules de Cardan) (on a posé : $j=e^{2i\pi/3}$ et $j^2=\bar{j}=e^{4i\pi/3}$) • si R < 0 : $\begin{cases} y_1=-2\sqrt{-p}.\cos\dfrac{\varphi}{3} \\ y_2=2\sqrt{-p}.\cos\dfrac{\varphi-\pi}{3} \\ y_3=2\sqrt{-p}.\cos\dfrac{\varphi+\pi}{3} \end{cases}$ avec $\cos\varphi=\dfrac{q}{\sqrt{-p^3}}$

2.8. Linéarisation des premiers polynômes trigonométriques

$\cos^2 x$	$\frac{1}{2}(1+\cos 2x)$
$\cos^3 x$	$\frac{1}{3}(3\cos x+\cos 3x)$
$\cos^4 x$	$\frac{1}{8}(3+4\cos 2x+\cos 4x)$
$\cos^5 x$	$\frac{1}{16}(10\cos x+5\cos 3x+\cos 5x)$
$\cos^6 x$	$\frac{1}{32}(10+15\cos 2x+6\cos 4x+\cos 6x)$
$\cos x\sin x$	$\frac{1}{2}\sin 2x$
$\cos^3 x\sin^3 x$	$\frac{1}{32}(3\sin 2x-\sin 6x)$
$\cos^2 x\sin x$	$\frac{1}{4}(\sin x+\sin 3x)$
$\cos^3 x\sin x$	$\frac{1}{8}(2\sin 2x+\sin 4x)$
$\cos^4 x\sin x$	$\frac{1}{16}(2\sin x+3\sin 3x+\sin 5x)$
$\cos^5 x\sin x$	$\frac{1}{32}(5\sin 2x+4\sin 4x+\sin 6x)$
$\cos^3 x\sin^2 x$	$\frac{1}{16}(2\cos x-\cos 3x-\cos 5x)$
$\cos^4 x\sin^2 x$	$\frac{1}{32}(2+\cos 2x-2\cos 4x-\cos 6x)$

$\sin^2 x$	$\frac{1}{2}(1-\cos 2x)$
$\sin^3 x$	$\frac{1}{4}(3\sin x-\sin 3x)$
$\sin^4 x$	$\frac{1}{8}(3-4\cos 2x+\cos 4x)$
$\sin^5 x$	$\frac{1}{16}(10\sin x-5\sin 3x+\sin 5x)$
$\sin^6 x$	$\frac{1}{32}(10-15\cos 2x+6\cos 4x-\cos 6x)$
$\cos^2 x\sin^2 x$	$\frac{1}{8}(1-\cos 4x)$
$\cos^4 x\sin^4 x$	$\frac{1}{128}(3-4\cos 4x+\cos 8x)$
$\cos x\sin^2 x$	$\frac{1}{4}(\cos x-\cos 3x)$
$\cos x\sin^3 x$	$\frac{1}{8}(2\sin 2x-\sin 4x)$
$\cos x\sin^4 x$	$\frac{1}{16}(2\cos x-3\cos 3x+\cos 5x)$
$\cos x\sin^5 x$	$\frac{1}{32}(5\sin 2x-4\sin 4x+\sin 6x)$
$\cos^2 x\sin^3 x$	$\frac{1}{16}(2\sin x+\sin 3x-\sin 5x)$
$\cos^2 x\sin^4 x$	$\frac{1}{32}(2-\cos 2x-2\cos 4x+\cos 6x)$

3. Calcul différentiel et intégral

3.1. Dérivées

f	f'
u^{α} (α constante)	$\alpha . u' . u^{\alpha-1}$
$\alpha = -1$: $\frac{1}{u}$	$\frac{-u'}{u^2}$
$\alpha = \frac{1}{2}$: $\sqrt{u}$	$\frac{u'}{2\sqrt{u}}$
$\ln u$	$\frac{u'}{u}$
e^u	$u' . e^u$
a^u (a > 0)	$u' . a^u . \ln a$
$\text{sh} u$	$u' . \text{ch} u$
$\text{ch} u$	$u' . \text{sh} u$
$u + v + w$	$u' + v' + w'$
$\frac{u}{v}$	$\frac{u'.v - u.v'}{v^2}$

f	f'
$\sin u$	$u' . \cos u$
$\cos u$	$-u' . \sin u$
$\tan u$	$u' . \left(1 + \tan^2 u\right) = \frac{u'}{\cos^2 u}$
$\text{Arc}\sin u$	$\frac{u'}{\sqrt{1-u^2}}$
$\text{Arc}\cos u$	$\frac{-u'}{\sqrt{1-u^2}}$
$\text{Arc}\tan u$	$\frac{u'}{1+u^2}$
$\text{Argsh} u$	$\frac{u'}{\sqrt{u^2+1}}$
$\text{Argch} u$	$\frac{u'}{\sqrt{u^2-1}}$
$u.v.w$	$u'.v.w + u.v'.w + u.v.w'$
u^v	$\left(\frac{u'}{u} v + v' . \ln u\right) u^v$

3.2. Développements en série de Taylor

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + x^n \varepsilon(x)$	$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ... + x^n \varepsilon(x)$
$\text{ch} x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ... + x^n \varepsilon(x)$	$\frac{1}{1+x} = 1 - x + x^2 - x^3 + ... + x^n \varepsilon(x)$
$\text{sh} x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + ... + x^n \varepsilon(x)$	$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{1}{4 \times 2!} x^2 + \frac{1 \times 3}{8 \times 3!} x^3 - \frac{1 \times 3 \times 5}{16 \times 4!} x^4 + ... + x^n \varepsilon(x)$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + x^n \varepsilon(x)$	$(1+x)^{\alpha} = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + ... + x^n \varepsilon(x)$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... + x^n \varepsilon(x)$	$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + x^{10} \varepsilon(x)$
$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ... + x^n \varepsilon(x)$	$\text{Arc}\tan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ... + x^n \varepsilon(x)$

3.3. Primitives

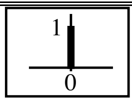
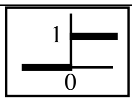
f	$\int f$
$x^{\alpha} \quad (\alpha \neq -1)$	$\dfrac{x^{\alpha+1}}{\alpha+1} + \mathrm{C}$
$\dfrac{1}{x}$	$\ln\lvert x\rvert + \mathrm{C}$
e^{x}	$e^{x} + \mathrm{C}$
$e^{ax}\cos bx$	$e^{ax}\dfrac{a\cos bx + b\sin bx}{a^2+b^2} + \mathrm{C}$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\mathrm{Arc}\sin\dfrac{x}{a} + \mathrm{C}$
ou bien $\dfrac{1}{\sqrt{a^2-x^2}}$	$-\mathrm{Arc}\cos\dfrac{x}{a} + \mathrm{C}$
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\mathrm{Arc}\tan\dfrac{x}{a} + \mathrm{C}$
$\sin x$	$-\cos x + \mathrm{C}$
$\cos x$	$\sin x + \mathrm{C}$
$\tan x$	$-\ln\lvert\cos x\rvert + \mathrm{C}$
$\mathrm{cotan}\,x$	$\ln\lvert\sin x\rvert + \mathrm{C}$
$\sin^2 x$	$\dfrac{1}{2}(x - \sin x\cos x) + \mathrm{C}$
$\cos^2 x$	$\dfrac{1}{2}(x + \sin x\cos x) + \mathrm{C}$
$\tan^2 x$	$\tan x - x + \mathrm{C}$
$\dfrac{1}{\sin x}$	$\ln\left\lvert\tan\dfrac{x}{2}\right\rvert + \mathrm{C}$
$\dfrac{1}{\cos x}$	$\ln\left\lvert\tan\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right\rvert + \mathrm{C}$
$\dfrac{1}{\sin^2 x}$	$-\mathrm{cotan}\,x + \mathrm{C}$
$\dfrac{1}{\cos^2 x}$	$\tan x + \mathrm{C}$

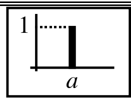
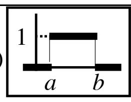
f	$\int f$
$\ln x$	$x\ln x - x + \mathrm{C}$
$\dfrac{1}{x\ln x}$	$\ln(\ln x) + \mathrm{C}$
$a^{x} \quad (a>0)$	$\dfrac{1}{\ln a}a^{x} + \mathrm{C}$
$e^{ax}\sin bx$	$e^{ax}\dfrac{-b\cos bx + a\sin bx}{a^2+b^2} + \mathrm{C}$
$\dfrac{1}{\sqrt{x^2+h}}$	$\ln\left\lvert x+\sqrt{x^2+h}\right\rvert + \mathrm{C}$
$\dfrac{1}{\sqrt{x^2+a^2}}$	$\mathrm{Arg\,sh}\dfrac{x}{a} + \mathrm{C}$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\mathrm{Arg\,ch}\dfrac{x}{a} + \mathrm{C}$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert + \mathrm{C}$
$\mathrm{sh}\,x$	$\mathrm{ch}\,x + \mathrm{C}$
$\mathrm{ch}\,x$	$\mathrm{sh}\,x + \mathrm{C}$
$\mathrm{th}\,x$	$\ln\mathrm{ch}\,x + \mathrm{C}$
$\coth x$	$\ln\lvert\mathrm{sh}\,x\rvert + \mathrm{C}$
$\mathrm{sh}^2 x$	$\dfrac{1}{2}(\mathrm{sh}\,x\,\mathrm{ch}\,x - x) + \mathrm{C}$
$\mathrm{ch}^2 x$	$\dfrac{1}{2}(\mathrm{sh}\,x\,\mathrm{ch}\,x + x) + \mathrm{C}$
$\mathrm{th}^2 x$	$x - \mathrm{th}\,x + \mathrm{C}$
$\dfrac{1}{\mathrm{sh}\,x}$	$\ln\left\lvert\mathrm{th}\,\dfrac{x}{2}\right\rvert + \mathrm{C}$
$\dfrac{1}{\mathrm{ch}\,x}$	$2\mathrm{Arc}\tan(e^{x}) + \mathrm{C}$
$\dfrac{1}{\mathrm{sh}^2 x}$	$\mathrm{th}\,x + \mathrm{C}$
$\dfrac{1}{\mathrm{ch}^2 x}$	$-\coth x + \mathrm{C}$

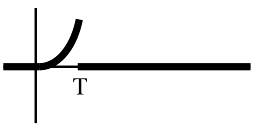
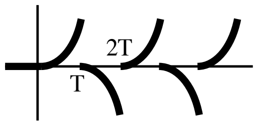
3.4. Équations différentielles

① linéaire, 1er ordre, sans second membre	$y'(x)+a(x)y(x)=0$	$y(x)=K.e^{-A(x)}$, où A est une primitive de a
② linéaire, 1er ordre, avec second membre	$y'(x)+a(x)y(x)=g(x)$	$y(x)=K(x).e^{-A(x)}$ où $\begin{cases} A \text{ est une primitive de } a \\ K \text{ est une primitive de } g(x).e^{A(x)} \end{cases}$
③ linéaire, 2nd ordre, coefficients constants, sans second membre	$ay''(x)+by'(x)+cy(x)=0$	$y(x)=A.\varphi_1(x)+B.\varphi_2(x)$ où : $\Delta=b^2-4ac$, r_1 et r_2 sont les racines du trinôme ar^2+br+c, et $\begin{cases} \text{si } \Delta>0 : r_1 \text{ et } r_2 \text{ sont réelles, } \varphi_1(x)=e^{r_1x}, \varphi_2(x)=e^{r_2x} \\ \text{si } \Delta=0 : r_1 = r_2,\ \varphi_1(x)=e^{r_1x}, \varphi_2(x)=xe^{r_1x} \\ \text{si } \Delta<0 : r_1 = \alpha+i\beta,\ r_2=\alpha-i\beta,\ \varphi_1(x)=\cos\beta x e^{\alpha.x}, \varphi_2(x)=\sin\beta x.e^{\alpha.x} \end{cases}$
④ linéaire, 2nd ordre, coefficients constants, avec second membre	$ay''(x)+by'(x)+cy(x)=g(x)$	$y(x)=A(x)\varphi_1(x)+B(x).\varphi_2(x)$ où $\begin{cases} \varphi_1 \text{ et } \varphi_2 \text{ sont obtenues comme en ③} \\ W(x)=\varphi_1(x)\varphi'_2(x)-\varphi'_1(x)\varphi_2(x)\neq 0 \quad (\text{Wronskien de } \varphi_1 \text{ et } \varphi_2) \\ A \text{ est une primitive de } \dfrac{-g(x)\varphi_2(x)}{W(x)} \\ B \text{ est une primitive de } \dfrac{g(x)\varphi_1(x)}{W(x)} \end{cases}$
⑤ type Bernoulli	$y'(x)+a(x)y(x)=g(x)y^m(x)$	$y(x)=z(x)^{1/(1-m)}$, où z est solution de l'éq. diff. de type ② : $\frac{1}{1-m}z'(x)+a(x)z(x)=g(x)$
⑥ type Ricatti	$y'(x)=a(x)+b(x)y(x)+c(x)y^2(x)$	$y(x)=y_1(x)+\frac{1}{z(x)}$, où $\begin{cases} y_1 \text{ est une solution particulière de l'éq. diff. initiale} \\ z \text{ est une solution quelconque de l'éq. diff. :} \\ z'(x)+\left[b(x)+2y_1(x)c(x)\right].z(x)=-c(x) \end{cases}$
⑦ type Euler-Cauchy	$ax^2y''(x)+bxy'(x)+cy(x)=0$	Poser $x=e^t$, l'éq. diff. devient du type ③ pour la variable t : $a\frac{d^2y}{dt^2}+(b-1)\frac{dy}{dt}+cy=0$

4. Transformation de Laplace

$f(t)$	$F(s)$
δ_0 	1
$Y(t)$ 	$\frac{1}{s}$
$t.Y(t)$	$\frac{1}{s^2}$
$t^n.Y(t)$	$\frac{n!}{s^{n+1}}$
$\sqrt{t}.Y(t)$	$\frac{\sqrt{\pi}}{2s\sqrt{s}}$
$\frac{1}{\sqrt{t}}Y(t)$	$\sqrt{\frac{\pi}{s}}$
$\cos\omega t.Y(t)$	$\frac{s}{s^2+\omega^2}$
$\mathrm{ch}\omega t.Y(t)$	$\frac{s}{s^2-\omega^2}$
$e^{-at}\cos\omega t.Y(t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\mathrm{t.cos}\omega t.Y(t)$	$\frac{s^2-\omega^2}{\left(s^2+\omega^2\right)^2}$
$\mathrm{t.ch}\omega t.Y(t)$	$\frac{s^2+\omega^2}{\left(s^2-\omega^2\right)^2}$
$\frac{\sin\omega t+\omega t.\cos\omega t}{2\omega}Y(t)$	$\frac{s^2}{\left(s^2+\omega^2\right)^2}$
$f(t-\tau)$	$e^{-s\tau}F(s)$
$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
$f'(t)$	$s.F(s)-f(0)$
$\frac{\partial f}{\partial t}(x,t)$	$s.F(x,s)-f(x,0)$
$\frac{\partial f}{\partial x}(x,t)$	$\frac{\partial F}{\partial x}(x,s)$

$f(t)$	$F(s)$
δ_a 	e^{-as}
$Y(t-a)-Y(t-b)$ 	$\frac{1}{s}\left(e^{-as}-e^{-bs}\right)$
$t^2.Y(t)$	$\frac{2}{s^3}$
$e^{-at}.Y(t)$	$\frac{1}{s+a}$
$t\sqrt{t}.Y(t)$	$\frac{3\sqrt{\pi}}{4s^2\sqrt{s}}$
$\frac{1}{t\sqrt{t}}Y(t)$	$-2\sqrt{\pi s}$
$\sin\omega t.Y(t)$	$\frac{\omega}{s^2+\omega^2}$
$\mathrm{sh}\omega t.Y(t)$	$\frac{\omega}{s^2-\omega^2}$
$e^{-at}\sin\omega t.Y(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\mathrm{t.sin}\omega t.Y(t)$	$\frac{2s\omega}{\left(s^2+\omega^2\right)^2}$
$\mathrm{t.sh}\omega t.Y(t)$	$\frac{2s\omega}{\left(s^2-\omega^2\right)^2}$
$\frac{\sin\omega t-\omega t.\cos\omega t}{2\omega}Y(t)$	$\frac{1}{\left(s^2+\omega^2\right)^2}$
$e^{-at}f(t)$	$F(s+a)$
$\int_0^t f(u)du$ (primitive de f qui s'annule en 0)	$\frac{F(s)}{s}$
$f''(t)$	$s^2.F(s)-s.f(0)-f'(0)$
$\frac{\partial^2 f}{\partial t^2}(x,t)$	$s^2.F(x,s)-s.f(x,0)-\frac{\partial f}{\partial t}(0)$
$\frac{\partial^2 f}{\partial x^2}(x,t)$	$\frac{\partial^2 F}{\partial x^2}(x,s)$

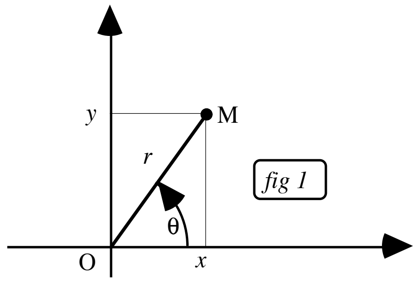
motif	fonction périodique	fonction alternée
		
$F_0(s)$	$\frac{F_0(s)}{1-e^{-sT}}$	$\frac{F_0(s)}{1+e^{-sT}}$

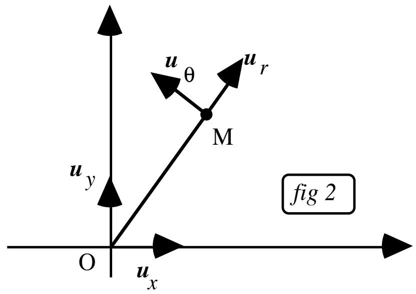
5. Systèmes de coordonnées
&
Opérateurs différentiels

f représente une fonction scalaire et $\vec{F}$ *une fonction vectorielle, c'est-à-dire :*

$$\begin{cases} f(x,y) \in \mathbb{R} \\ \vec{F}(x,y) = F_x(x,y).\vec{u}_x + F_y(x,y).\vec{u}_y \end{cases} \quad \textit{ou bien} \quad \begin{cases} f(x,y,z) \in \mathbb{R} \\ \vec{F}(x,y,z) = F_x(x,y,z).\vec{u}_x + F_y(x,y,z).\vec{u}_y + F_z(x,y,z).\vec{u}_z \end{cases}$$

5.1. Coordonnées polaires

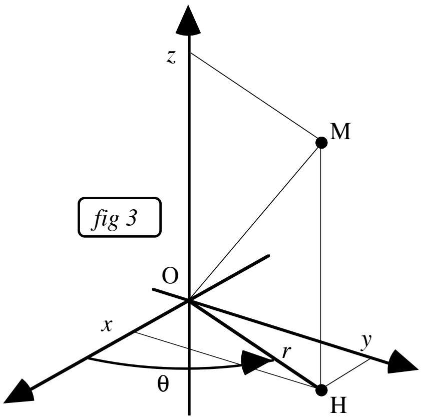


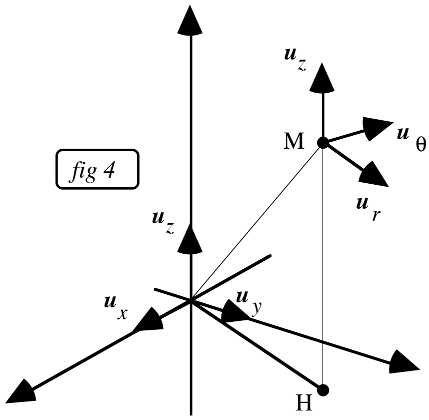


définition (fig 1)	changement de base (fig 2)	fonction vectorielle
$\begin{cases} x = r.\cos\theta \\ y = r.\sin\theta \end{cases}$ avec $r \geq 0$ et $\theta \in [0;2\pi[$	$\begin{cases} \vec{u}_x = \cos\theta.\vec{u}_r - \sin\theta.\vec{u}_\theta \\ \vec{u}_y = \sin\theta.\vec{u}_r + \cos\theta.\vec{u}_\theta \end{cases}$	$\begin{cases} F_x = \cos\theta.F_r - \sin\theta.F_\theta \\ F_y = \sin\theta.F_r + \cos\theta.F_\theta \end{cases}$

dérivées premières	$\begin{cases} \dfrac{\partial f}{\partial x} = \cos\theta.\dfrac{\partial f}{\partial r} - \dfrac{1}{r}\sin\theta.\dfrac{\partial f}{\partial \theta} \\ \dfrac{\partial f}{\partial y} = \sin\theta.\dfrac{\partial f}{\partial r} + \dfrac{1}{r}\cos\theta.\dfrac{\partial f}{\partial \theta} \end{cases}$
$dx.dy =$	$r.dr.d\theta$
$\overrightarrow{grad}(f) =$	$\dfrac{\partial f}{\partial r}.\vec{u}_r + \dfrac{1}{r}\dfrac{\partial f}{\partial \theta}.\vec{u}_\theta$
$\text{div}(\vec{F}) =$	$\dfrac{1}{r}\dfrac{\partial}{\partial r}(r.F_r) + \dfrac{1}{r}\dfrac{\partial F_\theta}{\partial \theta}$
$\Delta f =$	$\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial f}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 f}{\partial \theta^2}$

5.2. Coordonnées cylindriques

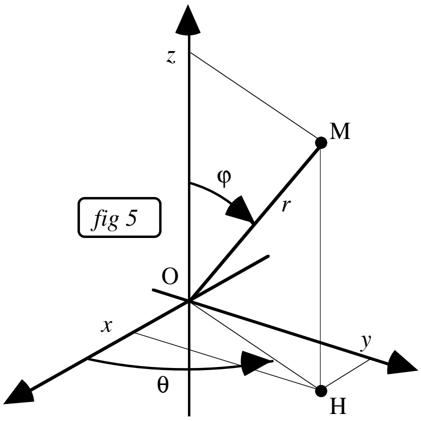


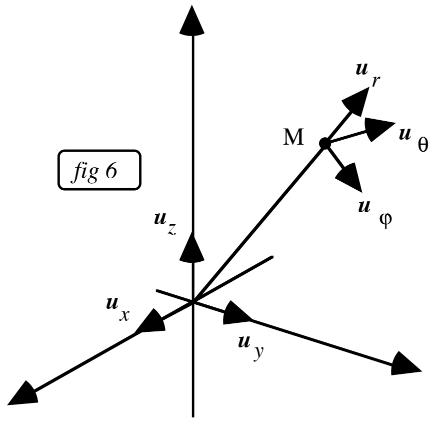


définition (fig 3)	changement de base (fig 4)	fonction vectorielle
$\begin{cases} x = r.\cos\theta \\ y = r.\sin\theta \quad \text{avec } r \geq 0 \text{ et } \theta \in [0;2\pi[\\ z = z \end{cases}$	$\begin{cases} \vec{u}_x = \cos\theta.\vec{u}_r - \sin\theta.\vec{u}_\theta \\ \vec{u}_y = \sin\theta.\vec{u}_r + \cos\theta.\vec{u}_\theta \\ \vec{u}_z = \vec{u}_z \end{cases}$	$\begin{cases} F_x = \cos\theta.F_r - \sin\theta.F_\theta \\ F_y = \sin\theta.F_r + \cos\theta.F_\theta \\ F_z = F_z \end{cases}$

dérivées premières	$\begin{cases} \dfrac{\partial f}{\partial x} = \cos\theta\dfrac{\partial f}{\partial r} - \dfrac{1}{r}\sin\theta\dfrac{\partial f}{\partial \theta} \\ \dfrac{\partial f}{\partial y} = \sin\theta\dfrac{\partial f}{\partial r} + \dfrac{1}{r}\cos\theta\dfrac{\partial f}{\partial \theta} \\ \dfrac{\partial f}{\partial z} = \dfrac{\partial f}{\partial z} \end{cases}$
$dx.dy.dz =$	$r.dr.d\theta.dz$
$\overrightarrow{grad}(f) =$	$\dfrac{\partial f}{\partial r}.\vec{u}_r + \dfrac{1}{r}\dfrac{\partial f}{\partial \theta}.\vec{u}_\theta + \dfrac{\partial f}{\partial z}\vec{u}_z$
$\text{div}(\vec{F}) =$	$\dfrac{1}{r}\dfrac{\partial}{\partial r}(r.F_r) + \dfrac{1}{r}\dfrac{\partial F_\theta}{\partial \theta} + \dfrac{\partial F_z}{\partial z}$
$\overrightarrow{rot}(\vec{F}) =$	$\left(\dfrac{1}{r}\dfrac{\partial F_z}{\partial} - \dfrac{\partial F_\theta}{\partial z}\right).\vec{u}_r + \left(\dfrac{\partial F_r}{\partial z} - \dfrac{\partial F_z}{\partial r}\right).\vec{u}_\theta + \left[\dfrac{1}{r}F_\theta + \left(\dfrac{\partial F_\theta}{\partial r} - \dfrac{1}{r}\dfrac{\partial F_r}{\partial \theta}\right)\right].\vec{u}_z$
$\Delta f =$	$\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial f}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 f}{\partial \theta^2} + \dfrac{\partial^2 f}{\partial z^2}$

5.3. Coordonnées sphériques





définition (fig 5)	changement de base (fig 6)	fonction vectorielle
$\begin{cases} x = r\sin\varphi\cos\theta \\ y = r\sin\varphi\sin\theta \\ z = r\cos\varphi \end{cases}$ avec $r \geq 0$, $\theta \in [0;2\pi[$ et $\varphi \in [0;\pi]$	$\begin{cases} \vec{u}_x = \cos\theta\sin\varphi.\vec{u}_r + \cos\theta\cos\varphi.\vec{u}_\varphi - \sin\theta.\vec{u}_\theta \\ \vec{u}_y = \sin\theta\sin\varphi.\vec{u}_r + \sin\theta\cos\varphi.\vec{u}_\varphi + \cos\theta.\vec{u}_\theta \\ \vec{u}_z = \cos\varphi.\vec{u}_r - \sin\varphi.\vec{u}_\varphi \end{cases}$	$\begin{cases} F_x = \cos\theta\sin\varphi.F_r + \cos\theta\cos\varphi.F_\varphi - \sin\theta.F_\theta \\ F_y = \sin\theta\sin\varphi.F_r + \sin\theta\cos\varphi.F_\varphi + \cos\theta.F_\theta \\ F_z = \cos\varphi.F_r - \sin\varphi.F_\varphi \end{cases}$

dérivées premières	$\begin{cases} \dfrac{\partial f}{\partial x} = \cos\theta\sin\varphi\dfrac{\partial f}{\partial r} + \dfrac{1}{r}\cos\theta\cos\varphi\dfrac{\partial f}{\partial\varphi} - \dfrac{1}{r\sin\varphi}\sin\theta\dfrac{\partial f}{\partial\theta} \\ \dfrac{\partial f}{\partial y} = \sin\theta\sin\varphi\dfrac{\partial f}{\partial r} + \dfrac{1}{r}\sin\theta\cos\varphi\dfrac{\partial f}{\partial\varphi} + \dfrac{1}{r\sin\varphi}\cos\theta\dfrac{\partial f}{\partial\theta} \\ \dfrac{\partial f}{\partial z} = \cos\varphi\dfrac{\partial f}{\partial r} - \dfrac{1}{r}\sin\varphi\dfrac{\partial f}{\partial\varphi} \end{cases}$
$dx.dy.dz =$	$r^2\sin\varphi.dr.d\theta.d\varphi$
$\overrightarrow{grad}(f) =$	$\dfrac{\partial f}{\partial r}.\vec{u}_r + \dfrac{1}{r}\dfrac{\partial f}{\partial\varphi}.\vec{u}_\varphi + \dfrac{1}{r\sin\varphi}\dfrac{\partial f}{\partial\theta}\vec{u}_\theta$
$div(\vec{F}) =$	$\dfrac{1}{r}\dfrac{\partial}{\partial r}(r.F_r) + \dfrac{1}{r}\dfrac{\partial F_\varphi}{\partial\varphi} + \dfrac{1}{r\sin\varphi}\dfrac{\partial F_\theta}{\partial\theta}$
$\overrightarrow{rot}(\vec{F}) =$	$\left[\dfrac{1}{r\tan\varphi}F_\theta + \dfrac{1}{r}\left(\dfrac{\partial F_\theta}{\partial\varphi} - \dfrac{1}{\sin\varphi}\dfrac{\partial F_\varphi}{\partial\theta}\right)\right].\vec{u}_r + \left[\dfrac{-1}{r}F_\theta + \left(\dfrac{1}{r\sin\varphi}\dfrac{\partial F_r}{\partial\theta} - \dfrac{\partial F_\theta}{\partial r}\right)\right].\vec{u}_\varphi + \left[\dfrac{1}{r}F_\varphi + \left(\dfrac{\partial F_\varphi}{\partial r} - \dfrac{1}{r}\dfrac{\partial F_r}{\partial\varphi}\right)\right].\vec{u}_\theta$
$\Delta f =$	$\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial f}{\partial r}\right) + \dfrac{1}{r^2\sin^2\varphi}\dfrac{\partial}{\partial\varphi}\left(\sin^2\varphi\dfrac{\partial f}{\partial\varphi}\right) + \dfrac{1}{r^2\sin^2\varphi}\dfrac{\partial^2 f}{\partial\theta^2}$