CHAPITRE 6

VALIDATION WITH THE DSO METHOD

The primary objective of this chapter is intended to make use of DSO Algorithm 2 that combines GA's and the discrete stochastic optimization theory in order to compare and further to validate the IV-wavelet-GA approach which has been established in the previous chapters, but would need a rigorous mathematical proof in order to bridge such a traditional short fall. The key element in such a validation process is to transform the IV time series data set into a cross-sectional one. The validation process would then involve the forecast of the one-step-ahead directions and ranges of the IV series derived from the S&P500 index. The results hence obtained are expected to have a similar nature to what have been achieved in Chapter 5 *i.e.,* the accuracy in forecasting the S&P500 volatility is expected to be in the same order as that was derived in forecasting the S&P100. In Section 6.1, 6.2 and 6.3, we have presented the forecast for the S&P500 2004 volatility by means of GARCH(1,1), compared critically the hence obtained results with those derived from applying the DSO method and then attempted to validate those DSO results with the bootstrapping tests. In Section 6.4 and 6.5 a general discussion on the approach taken is presented with respect to its limitations and potential applications.

6.1 Forecast with GARCH(1,1) method

The following section describes the computational phases involved in this GARCH forecast. The data set S&P500 index from Olsen & Associates Inc. is processed employing the Matlab applications.

The 2002-2003 and 2002 index data are used to train the algorithm, whereas the Jan. 2-Oct. 21, 2004 data are used as the out of sample data to test the derived rules. The training data sets are first processed to derive the parameters for the GARCH(1,1) model. The data in 2004 are then fed to the resulting GARCH(1,1) model in order to obtain the one-day-ahead forecasted IV values. The same data are then used to compute the (realized) IV values for the 2004 data set. Both groups of results are normalized to the respective logarithmic means and then converted into values of 1, 2, 3 and 4 according to their amplitudes, similar to the pre-processing described in the previous sections where GA's are used. The accuracy of forecasting is computed based on the comparison between the two groups of the converted data. The one-day-ahead forecasting accuracy of 2004 based on training sets corresponding to 2002-2003 and 2003 alone is thus obtained and is shown in the following table:

Table VI

The one-day-ahead GARCH forecasting accuracy for the 2004 S&P500 daily data

Training Period	% Accuracy
2003	43.9
2002—2003	35.6

They agree moderately well with those results derived in a number of contemporary GARCH as well as IV studies (Andersen, 1998, 2001), and they validated our data pre-processing procedure, but are markedly lower than those achieved by using the GA-DSO method proposed and developed in this research work. The GA-DSO method can thus be shown to be superior to the GARCH approach simply because it takes more historical patterns, linear or nonlinear, into consideration for forecasting purposes. More pertinent details regarding the comparison are given in the following sections.

6.2 Data Processing with S&P500

To apply the DSO method for the validation purpose, the same IV time series S&P100 derived in Section 5.1 is converted into wavelet coefficients by following the same procedure as those discussed in Sections 5.2 and 5.3. Based on the entropy level, coefficients at node (2, 0) in the wavelet tree are selected for the Algorithm 2 to process with Matlab applications. To form the initial population, 100 groups of rules are randomly generated with each consisting of 100 rules. GA operations such as selection, cross-over and mutation are then applied to find ultimately the best 100 rules among all rules regardless of which groups they belong. Initially, up to 500 generations have been imposed as the stopping criterion according to the experience gained in previous tests, but it was soon found that results tend to converge after 75 generations. Fourteen runs of tests each with 75 generations have been performed. For the sake of brevity, results of the last five are plotted in Fig. 7 as shown in the following.

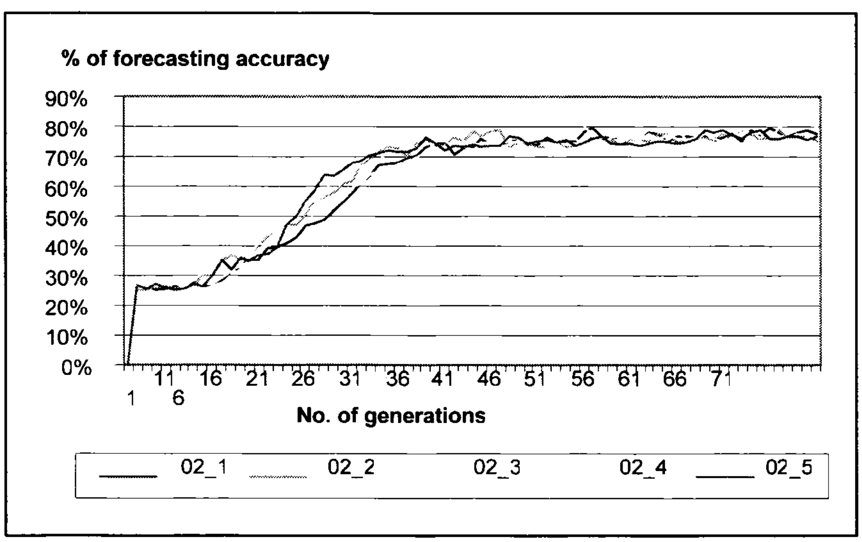

Figure 7 Daily 2003 S&P100 forecasting accuracy with 2002 data as training set

The data set S&P500 index from Olson & Associates Inc. is then processed. The price data in 2003 is used for training purpose, while the 2004 data are used to test the optimized rules. The same procedure is then repeated with the 2002 – 2003 data as the training set. Again, tests with up to 500 generations have shown that results tend to converge after 75 generations. Please refer to Fig. 8 for a display of selected results of the last few runs for the two sets of tests.

One could easily observe that the forecasting accuracy reaches well above 70% for both sets of tests. One advantage of using the DSOM algorithm 2 is its fast convergence due mainly to the usage of far larger amount of memory to record all the rule history. In contrast, simple GA's discussed in Chapter 5 made used of great deal of loops generation over generation. It is interesting however, the forecasting accuracy for the 2003 data set is generally higher than those derived based on 2002-2003 data sets *e.g.*, above 80% versus above 70%. Such a phenomenon actually agrees well with the widely documented observation in volatility time series – the lesson being that the more recent events carry more weight to the current reading than those further back in the past.

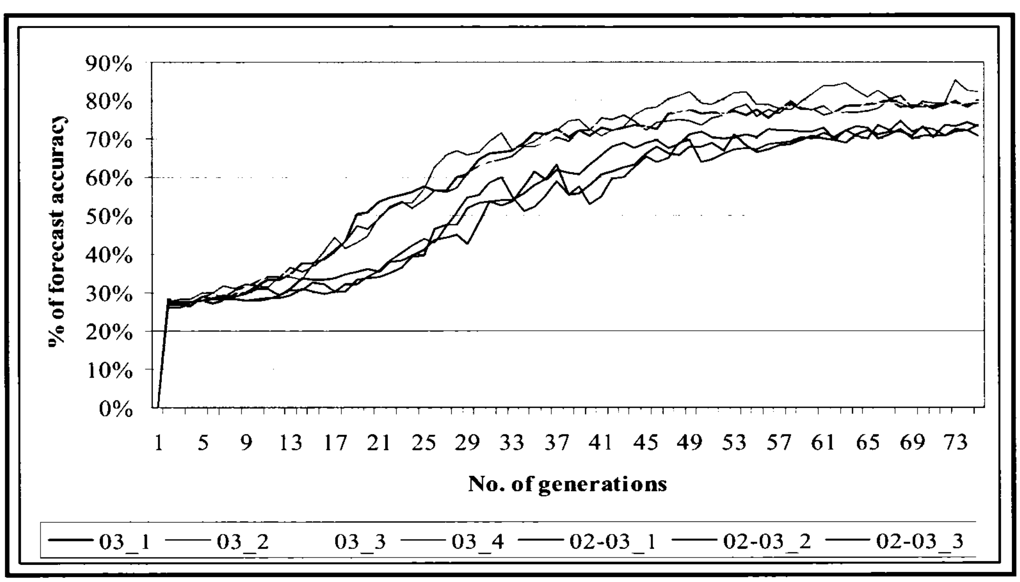

Figure 8 Daily 2004 S&P500 forecasting accuracy trained by 2002-3 and 2002 data

6.3 Statistical Validation

A confidence interval is usually needed to depict an accurate estimate of a random variable, which is supposed to include the true value of the variable with a specified probability. Bootstrapping provides a ready and reliable way to construct such a confidence interval that does not depend on the asymptotic normality assumption. This is important when the population consists of just a few observations. We have used Carpenter and Bithell's (2001) approach to process the two 14 forecasts that were respectively derived from the one-year data of S&P100 and S&P500. The bootstrapping samples are chosen to be 999, confidence level to be 0.05 and statistical function to be MEAN. The results are 0.7734 [0.7659, 0.7794] and 0.8144[0.8067, 0.8233] respectively. The 0.7734 forecasting accuracy of the S&P100 IV matches the result that was achieved with the GA alone as shown in Chapter 5. However, the current 75 generations versus then 1000 generations represent more than 10 folds reduction of CPU time.

6.4 Discussion of the Results

In comparison with the GA operation in Ma *et al.* (2004*b*) which repetitively loop through Step 1 to 3 in Algorithm1 without keeping all tested rules in memory, the current GA-DSO approach has the following advantages:

a) Evaluate fewer rules per iteration by avoiding re-tests of the rules stored in the memory, thus resulting in greater computation efficiency;
b) Use tournament selection method instead of the ranking method, hence minimizing the risk of early convergence;

c) Take all historical rules into account and not just those retained in the last generation while optimizing their respective final fitness values, hence improving forecasting accuracy.

It is therefore, obvious that results obtained in this chapter validate the following two claims:

a) The DSO procedure produces forecasting results at the same or higher accuracy levels in comparison with those obtained by using GA alone
b) The DSO procedure that incorporated GA produces results by processing IV's of S&P100, which are consistent with those by processing IV's of S&P500.

In other words, the current set of experimental tests coupled with the analysis developed in Chapter 4 confirm that the data conversion of a regular time series into the 4-lag recursive data set based on the TSDM framework could indeed be treated as a Markov chain. And the application of GA could indeed be substantiated by the DSO theory. For those who intend to try a test drive based on the approach established here, more numerical tests will definitely help them obtain a better grasp on the methodology and the claims made herein. However, the experimental nature of EA's thus dictates that no two tests would yield identical results, whereas slightly different results would not materially undermine the conclusions that we have reached here.

Another advantage of the current approach is characterized by the ease of assessment on the result quality, in which a direct comparison of percentage accuracy would suffice. This would have created a problem to most traditional statistical models. First, there are several quality measures to choose from *e.g.*, as simple as error measurement of Mean Absolute Error (MAE) or MSE. Second, bias of both realized volatility (the target variable) and volatility forecasts appears if the return intervals chosen are too small due to the microstructure of the market. Volatility forecast tests are affected by this bias. A treatment of the bias is almost inevitable when designing volatility forecasting models and tests based on high-frequency returns over intervals of less than an hour (Corsi *et al.* 2001). Due to these technical difficulties, there is not yet a comprehensive study of high-frequency volatility forecasts with particular focus on their qualities (Dacorogna et al. 2001).

One of the reasons for Andratottir's global optimization method adopted here to be superior to the local optimization method is attributable to the nature of the converted data sets and the objectives of the current tests (Andradottir, 1999). Assume that $\theta_M^{*(1)}$ and $\theta_M^{*(2)}$ are ranked top one and two rules among the top 100 rules after the terminal generation. Based on the initial assumption, $\theta_M^{*(1)}$ will be the global optimal solution of the problem, while $\theta_M^{*(2)}$ could be a solution that locates as a state next to $\theta_M^{*(1)}$. Even in such a case, the importance of $\theta_M^{*(2)}$ is not diminished. This is mainly because the prediction accuracy calculated here is based on those qualified rules and not the absolute number of data points in a data set. In fact, the discrete nature of the rules dictates that different rules characterize different patterns of data points. Our objective is to find those patterns that match the four lags in the IV data set for the forecast of the fifth point. Rules corresponding to better fitness values would naturally have better chance to achieve higher forecasting accuracy in the entire data set regardless of the relation among rules if there is any. Therefore, rules derived from the global optimization method have a better generality than those derived from the local optimization method. For further proof of such a claim, it may be worthwhile to determine the effect of using different number of lags in forming rules and different values of *a, b, c* and *d* in classifying the IV data ranges on the prediction accuracy. Upon implementing these procedures, we anticipate that the current methodology would become quite complete and ready to support a commercial software package for the use of general risk forecasting in a wider industrial fields. However, these tasks are beyond the objective of this thesis research and are recommended for future expansion of our concepts.

6.5 Application of the Results

6.5.1 Volatility – from an Option Trading Perspective

The Black-Scholes model indicates that the price of an option is a function of the stock price, exercise price, risk-free rate, time to expiration, volatility, and any dividends on the stock over the life of the option. Of these six variables, the stock price, exercise price, and time to expiration are easily observable. Hence, one could easily measure these without introducing any appreciable error. The risk-free rate is largely observable, and its impact is small. The dividends are not observable, but they are not too difficult to measure accurately. The volatility, however, is almost completely unobservable. And this explains why the implied volatility in the Black-Scholes model is more or less a catch-all term, capturing whatever variables are missing, as well as the possibility that the model is improperly specified or blatantly wrong.

The Black-Scholes model is a critical component in the modern option pricing theory, but it tells us nothing about why anyone would hold an option. The very fact that it ignores the stochastic nature of the volatility variable means that any option serves as well as any other option and it therefore, cannot motivate the holding of options. In reality some options are more desirable than others. Whatever factors that motivate the holding of options are simply not captured in the Black-Scholes model. Hence, these factors show up hidden within the implied volatility.

Other researchers believe that the volatility smile as defined in Appendix 1 reflects stochastic volatility. Volatility is surely not constant as assumed in the Black-Scholes model. If volatility is stochastic, researchers argue that the smile reflects the failure of the Black-Scholes model to capture the random nature of volatility. Others argue that the Black-Scholes model, which assumes that stock prices fluctuate in a smooth and continuous manner, fails to capture the true nature of stock price movements, which are observed to have discrete jumps. However, practitioners seem capable of operating in a world of volatility smiles. They even use the smile to simplify how they trade. For example, they oftentimes quote option prices not in terms of the actual price but in terms of the implied volatility. More specifically, a dealer might indicate an intention to sell the January 36 call by quoting a volatility of 25.92. This statement is interpreted to mean that the actual price is derived from the Black-Scholes model using a volatility of 25.92. Assuming agreement on the dividends and risk-free rate, such a quote for this option would lead to a price of $1.85. By quoting prices this way, traders can immediately see which options are truly more expensive, that is, after accounting for moneyness (how much the security price is in, at or out of the strike price), time to expiration, and whether the option is a call or a put. The phenomenon of different implied volatility values for the same security simply reflects another aspect of the stochastic nature of volatility. It is obvious that volatility has to be evaluated for individual cases in most of the practical applications. The methodology established in this thesis help analysts determine values of volatility for different time horizons at a higher confidence level and at a more efficient speed. Put differently, it helps confirm the stochastic nature of the volatility of equity indices – volatility of volatility. The proposed approach could deal with different volatility regardless of its distributions or other random nature such as the smile effect. The recently available option chain data enables analysts to compile the implied volatility series for a selected security. The IV-wavelet-EA method would process the historical time series as described above and would provide implied volatility forecasts as required. More details could be found in the subsequent sections in this chapter about the concept of volatility curve.

6.5.2 Potential Applications of the Proposed Methodology

Volatility estimation is in the heart of option valuation for securities such as equity, indices and fixed-income. As a result, a large and growing body of literature that emerges recently in this area has proposed a variety of approaches. Their two primary limitations are firstly the contemporary estimation techniques do not allow inference of the volatility of the underlying security's price movement in an arbitrary term. This restriction might not be a concern for applications targeted at real-time security trading. However, since one usually relies on a variable time-scale in the formulation of stochastic programming (*e.g.* short time-steps for the near terms and much longer time-steps for the longer terms) in strategic risk management applications, the prevalent estimation techniques may not be suitable for users to cope with wider time horizons *e.g.,* banks, hedge fund management firms and other major investment institutions. The second limitation is that the prevalent approaches require an *a priori* assumption of a particular functional form for the estimated volatility curve, which is undesirable under most practical circumstances (Marti, 2004).

For evaluating different types of securities particularly fixed-income securities and equity options that have expiration dates, besides the observable quantities such as the term structure of interest rates (yield curve), one would also need the term structure of volatility. Like the yield curve, the term structure of volatility describes the relationship between the term to expiration of the equity options and their corresponding volatility (*i.e.* the volatility of 3-day, 6-days, 1-month, *etc.*).

Extended from the discussion thus far, it becomes apparent that another advantage of current IV-wavelet-GA approach is that by incorporating the wavelet transform (Ma *et al.*, 2004*b*) one could easily construct a yield-curve-like volatility curve. Conversely, she could use it as a guideline to determine the forecasting horizon based on the values of entropy of the respective wavelet coefficients. This will be more reasonable than arbitrarily selecting the forecast horizon based on analysts' experience or requirements as used in other prevalent approaches. With an improved prediction of volatility, besides trading options we could also trade volatility itself. One way to do so would be to use the newly formed VIX Futures in the CFE (Chicago Futures Exchange), namely VXB. Refer to APPENDIX 2 for details.

6.5.3 Applications on a Winder Perspective

In a wider perspective, risk management as a field has evolved and expanded, more so in the recent years. Few years ago it covered credit, individual security and market risk. Today, it also covers operational risk, fraud risk, and several other dimensions. Based on a survey conducted in the SAS Institute, all of the interviewees note that the underlying growth in the number and volume of financial risk-management securities and contracts are the fundamental driving force for continued growth in this field. One of the most popular approaches to risk measurement is by calculating what is known as an institution's "Value at Risk" (VaR), an estimation of likely losses that could arise from changes in market prices. More specifically, it is the money-loss in a portfolio that is expected to occur over a pre-determined horizon and with a pre-determined degree of confidence. Brooks paper (2003) illustrated the directly proportional relationship between volatility forecast and financial risk management *e.g.*, the large role played by the time-varying volatilities in minimizing VaR. The application of the current forecasting strategy is therefore, not limited to financial data forecasting or security trading, but have broader implications. For example, banks could use the technique to detect credit card frauds by following the spending patterns of their customers. Lending organizations could evaluate the bankruptcy risk of certain types of firms based on the selected criteria. Insurance companies could be in a better position to determine the risk of certain group of individuals by better predicting their behaviour. In short, the strategy of employing multiple engineering techniques to simplify and further to analyze time series could have a great potential in the data mining field.

In summary, we have incorporated Povinelli's (1999) TSDM framework with carefully selected engineering techniques such as wavelet transforms and GA/GP, substantiated the operation with DSO and consequently formulated a systematic approach to achieve markedly improved effectiveness and efficiency in forecasting volatility, which was not available till now. Volatility modeling and forecasting are one of the major obstacles in contemporary computational finance and econometrics. Using a data mining/EA approach to help effectively solve such a problem should be considered a meaningful, timely and important contribution in the field.