CHAPTER 2

VOLTAGE DROP ANALYSIS

2.1 Introduction

This chapter is concerned with maintaining the voltage profile of a system by means of reactive power management. The mechanism causing voltage drop across the transmission line and the remedy action to cancel voltage drop is identified. Of the various test scenarios that have been examined the radial system has been chosen due to variety of reasons: 1) it gives clear insight into the mechanism of the voltage drop and the voltage boost; 2) every power system, regardless of its topology, seen from the point of coupling with power delivery substation, is seen as a radial system since it can be modeled as simple Thévenin equivalent circuit; 3) the radial systems are known to have lowest reliability and highest vulnerability and 4) the radial systems are most likely to develop voltage problems.

The chapter begins with a discussion of some basic concepts related to power transmission over the line leading toward identification of voltage drop causes and action to remedy it. Afterwards, an analysis of a non compensated line is undertaken followed by the effect of power factor correction of the load and then the analysis of a compensated line where the compensation is on the receiving-end high-voltage buses or low-voltage buses is effected. These analytical treatment of voltage regulation is the key to understanding the focus of this thesis and the conclusions which follow.

2.2 Phenomenology of Power Transfer

The power industry offers to its clients energy in the form of the electricity. The voltage and current are principal entities the industry operates with. The potential difference (voltage) is created in power plants using magnetic forces, transferred over the line and offered to consumes. Physical significance of potential difference is possibility to do a work by electrical forces between the points the potential difference exist. This work consists in moving free charges through conducting medium between two points the difference of potential is imposed (conduction current), allowing conversion of electrical energy performance. The work is energy in conversion process. The work effected by electrical forces in moving a charge q between points P_1 and P_2 the difference of the potential U exist is:

$$W = \int_{P_1}^{P_2} \vec{F} d\vec{l} = q \int_{P_1}^{P_2} \vec{E} d\vec{l} = qU \tag{2.1}$$

where U [V] is voltage (potential difference) between point P_1 and P_2. and E(V/m) is electrical field in conductor and q [C] is one charge carrier that participate in electrical current. When consumer load connected on utilities output within points the potential difference exists, it draws a current. It can be said that current is medium for energy conversion. The conduction current is drift motion of the free charges (free electrons), under the influence of the electrical forces (Coulomb's forces) and free charges in solids are consequence of metallic bonds.

This potential difference (voltage) provided by utility is created in magnetic field of generator using magnetic forces to redistribute free charges in the conductor, the armature of the generator is made from. The magnetic force per unit charge *q* in motion is given by:

$$\frac{\vec{F}_m}{q} = \vec{v} \times \vec{B} \tag{2.2}$$

where **B** is magnetic flux density created by excitation circuit and ferromagnetic material the generator rotor and stator are made of, and ***v*** is a velocity armature conductors move(in fact the conductors are stationary but field created by excitation circuit is moving). Under the influence of the magnetic forces, free charges are drifted toward one end of the conductor living the other end positively charged. The separation of the charges create electric field and potential difference. The electric forces balance magnetic forces according to:

$$\vec{F}_m + \vec{F}_e = q\left(\vec{v} \times \vec{B} + \vec{E}\right) \tag{2.3}$$

The potential difference created between two points is given as circulation of electric field between this two point act as the source of *emf* according:

$$\int_{P_1}^{P_2} \vec{E} d\vec{l} = V_2 - V_1 = U = -\int_{P_1}^{P_2} \left(\vec{v} \times \vec{B}\right) d\vec{l} \tag{2.4}$$

The equation (2.4) is referred as a motional *emf*. The generator of *emf* is number of conducting loop in rotating magnetic field, connected in series so that potential difference, obtained using magnetic forces to separate free charges, is added. Due to sinusoidal distribution of magnetic field in the generator and the generator geometry, the potential difference obtained pulsates following sinusoidal law. This potential difference is offered at distribution feeders to consumers. When consumer is connected on potential difference with its load, it draws current. The current flowing through the armature of generator creates its magnetic field (armature reaction). Interaction of the excitation magnetic fields and current through the armature (armature magnetic field) produces mechanical forces opposing to rotation according to:

$$d\vec{F} = I d\vec{l} \times \vec{B} \tag{2.5}$$

and mechanical work has to be invested to continue to rotate generator rotor to support potential difference that results in current flow. It is a process of conversion of mechanical energy into electrical energy.

The potential difference offered on distribution feeders has to be transferred through the line or, as it is said, line has to be energized or charged to function. As the potential difference pulsate, line is continuously charged and discharged. The charging of the line can be seen as charging current. As soon as difference of potential exist (charge is deposed) between two lines or between line and ground, the electric field exists surrounding the line suggesting that line can be modeled as capacitance per unit of length and current i(t) is charging current as shown in Fig.2. As the voltage pulsate, electromagnetic field surrounding the line pulsates too. The charging energy is provided by the source (generator) and the charging energy is continuously exchanged between electromagnetic field surrounding the line and generator magnetic field. The charging current is medium for the transfer of charging energy. The line charging energy can be termed reactive because it is not dissipated but exchanged between line and generator. It is consistent with the fact that, when generator has to supply reactive power the excitation current has to be increased in order to boost magnetic field. The reaction of the line against voltage changes is capacitive current:

$$i_C = C\frac{dU}{dt} \tag{2.6}$$

The line opposes to voltage changes. To change the potential difference, the charging of the line has to be changed first resulting in current that lead to voltage. During the line charging, charging current leads the voltage by 90 degrees as indicated in (2.6). Whether loaded or not, line has to be charged to function and charging energy has to be supplied. The charging energy depends on line length and voltage level.

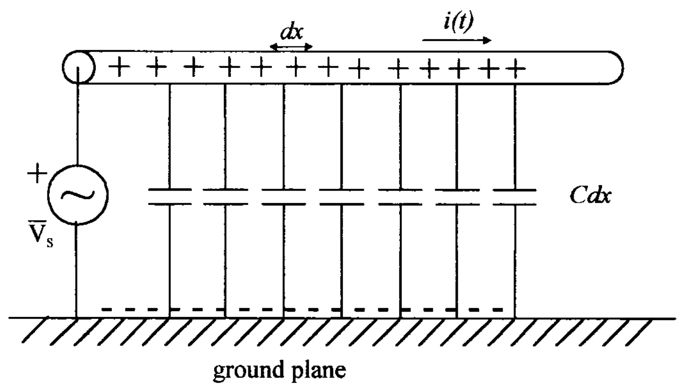


Figure 2 Open circuit transmission line during charging

When the line is loaded as shown in Fig.3, the load draws pulsating sinusoidal current that superpose to charging current. The pulsating current will cause pulsating magnetic field passing through the loop between two conductor (or conductor and ground) as illustrated in Fig.3.

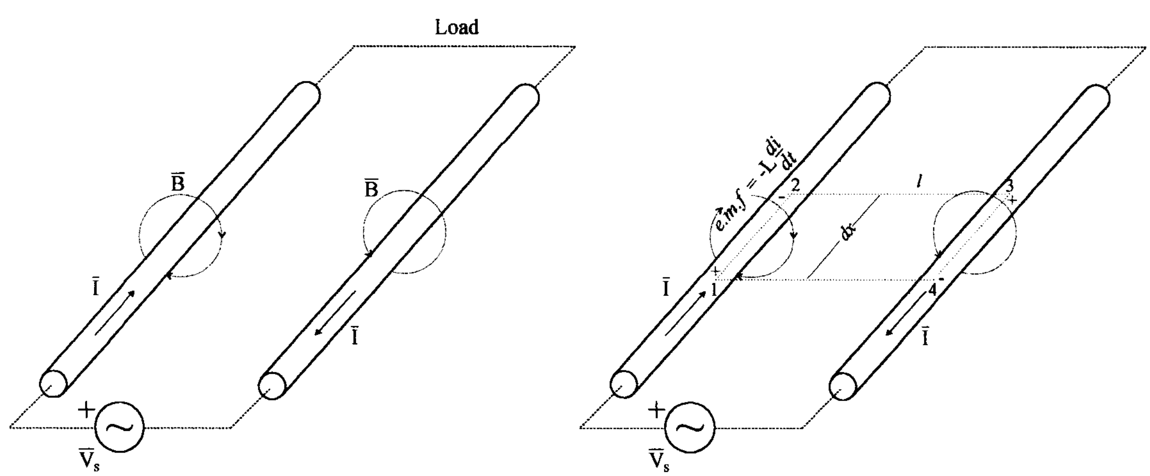


Figure 3 Simplified representation of transmission line

The pulsating magnetic field, in accordance with Faraday-Lenz's law, induce electric field opposing to its cause:

$$\oint_l \vec{E}d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{S} = -\frac{d}{dt}\int_S \vec{B}d\vec{S} = -\frac{d\Phi}{dt} = -\frac{d\Phi}{di}\frac{di}{dt} = -L\frac{di}{dt}[V] \tag{2.7}$$

It is possible to calculate *emf* induced along the line considering imaginary closed contour *l* along the line that is shown dotted on Fig.2.b. The circulation of vector **E** along the closed contour *l* is proportional to negative rate of increase of the magnetic flux linking the circuit. The induce *emf* along the contour *l* have the polarity that opposes to current change. The induced emf superpose to *emf* imposed by the source causing phase shift of voltage along the line. As it can be seen from (2.7), the induced *emf* can be expressed using self-inductance L of the line suggesting that infinitesimal lent of the line dx can be seen as inductance per unit of length. The self inductance L is defined as ratio of the magnetic flux linkage in the loop itself and current that created it. As the line is made from nonmagnetic material it is justified to assume that $\frac{d\Phi}{di} = \frac{\Phi}{I} = L$ where $L=2L_i+L_e$.where $L_i = \frac{\mu_0}{8\pi}\left[\frac{H}{m}\right]$ is internal inductance per unit length of the conductor and L_e is the external inductance per unit length that depends on geometry of the conductor dispositions. As the flux linkage depend on surface between two lines, and the line radius, the line inductance L is lower if the spacing between the conductors is lower and the radius is higher. The spacing between conductors is limited with permitivity of the dielectric between the conductors to prevent flashover. So every differential lent dx of lossless line can be represented using inductance and capacitance per line length as shown in Fig.4.

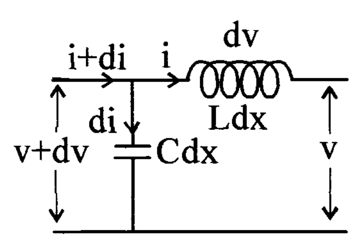


Figure 4 Transmission line model for lossless line

Considering open line with charging current only, at every point of the line charging current leads voltage by 90 degrees. The charging current creates pulsating magnetic field around the line. The pulsating field induces emf in accordance with Faraday-Lenz law:

$$emf = -L\frac{di_C}{dt} \tag{2.8}$$

Combining (2.6) and (2.8) gives:

$$emf = -L\frac{di_C}{dt} = -LC\frac{d^2U}{dt} \tag{2.9}$$

As voltage imposed by the source is sinusoidal:

$$U(t) = U_{\max}\cos\omega t \tag{2.10}$$

$$emf = LC\omega^2 U_{\max}\cos\omega t \tag{2.11}$$

It can be seen that (2.10) and (2.11) are in phase. At every point of the line, the induced emf due to charging current is in phase with emf generated by the source raising the voltage along the line. With increase in loading of the line, the load current increases superposing the charging current. Supposing the resistive load, load current is in phase with voltage:

$$i_l(t) = \frac{U(t)}{R} = \frac{U_{\max}}{R}\cos\omega t \tag{2.12}$$

$$i(t) = i_C(t) + i_l(t) = C\frac{dU}{dt} + \frac{U(t)}{R} = -CU_{\max}\omega\sin\omega t + \frac{U_{\max}}{R}\cos\omega t \tag{2.13}$$

With increase in load, pulsating magnetic field created by load current increases too (R decreases) together with induced *emf* along the line. Induced *emf* due to load current is added to induced *emf* due to pulsating charging current as:

$$emf = -L\frac{di}{dt} = -L\frac{d(i_C + i_l)}{dt} = -LC\frac{d^2U}{dt^2} - \frac{1}{R}\frac{dU(t)}{dt} = \\ = CLU_{\max}\omega^2\cos\omega t + \omega L\frac{U_{\max}}{R}\sin\omega t \tag{2.14}$$

The first term on the right side of equation (2.14) is in phase with source voltage along the line and is responsible for voltage boost and the second term on the right side of equation (2.14) is responsible for voltage drop because it is phase shifted by 90 degrees. With increase in loading of the line, the term responsible for voltage drop increases while term providing voltage boost is constant. It depends on charging current and charging current depends on voltage level and line parameters.

For the short line, the charging current is small. Therefore the term providing voltage boost can be neglected and term responsible for voltage drop is predominant. It is in accordance with line modeled as series inductance. Long lines require higher charging current. The induced *emf* due to charging current add along the line leading to increase in voltage along the line resulting in highest voltage at line end (Ferranti effect). It shows that voltage profile along the line is function of spatial coordinate as well as a function of time. It is in accordance with wave equation for voltage and current that can be deduced from Fig.4

$$\frac{\partial^2 u}{\partial x^2} = LC\frac{\partial^2 u}{\partial t^2} \tag{2.15}$$

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} \tag{2.16}$$

The (2.15) and (2.16) are partial differential equation. The solution of the wave equation for voltage and current (D'Alambert solution) is in the form:

$$\begin{aligned} u(x,t) &= u(x+ct) + u(x-ct) \\ i(x,t) &= i(x+ct) + i(x-ct) \end{aligned} \tag{2.17}$$

where $c^2 = \frac{1}{\sqrt{LC}}$ is velocity of propagation of the wave, L is inductance per unit of length and C is capacitance per unit of length. The solution (2.17) represents forward and backward traveling waves.

When the line is naturally loaded (matched-or loaded with natural impedance) the voltage and the current are in phase along the line. In this case, only forward wave exist indicating the fact that energy is transferred only in one direction, from the source towards the load.

Based on the above discussion the following explanation can be given. For functioning, the transmission line has to be charged. To charge the line, the work has to be done by the source of *emf* (generator) and this work is converted into the energy stored into the electromagnetic field surrounding the charged line. This energy is given by:

$$W = W_e + W_m = \frac{1}{2} \int_{space} \varepsilon E^2 dv + \frac{1}{2} \int_{space} \frac{B^2}{\mu} \tag{2.18}$$

where W_e is energy stored into the electric component of electromagnetic field and W_m is energy stored in magnetic component of electromagnetic field.

When energized with sinusoidal voltage, line is constantly charged and discharged. The intensity of electromagnetic field surrounding the line pulsates with line voltage and current, and the energy stored in electromagnetic field pulsates too. The charging energy is provided and absorbed by the source and transferred along the line and back due to change in voltage polarity. This energy is transferred through the line using the charging current as a medium. When the load is connected at the end of the line it draws the current, and the current creates magnetic field in the space surrounding the conductor superposing to magnetic field created by charging current. The magnetic field is pulsating (sinusoidal) because the current is pulsating and the magnetic field the energy is stored in. Due to pulsating magnetic field, electric field is induced in the space surrounding the line creating voltage drop along the line length according to Faraday-Lenz law. This voltage drop is sinusoidal, phase shifted compared to voltage imposed by the source and it superpose with voltage imposed by the source resulting in voltage that is changed in magnitude and phase. When the line is naturally loaded, the charging energy, ones supplied by the source, does not have to be exchanged between the line and source but rather is locally exchanged in electromagnetic field surrounding the line. Or, more precisely, the charging energy is locally exchanged between component of *electromagnetic* field surrounding the line that is consequence of charging current and the component of *electromagnetic* field that is consequence of load current. Or the voltage boost created by charging current is canceled by the voltage drop created by load current. It is worth noting that in both cases, voltage drop and voltage boost are consequence of phenomenology of Faraday-Lenz law. In this case, it can be said that line compensate itself.

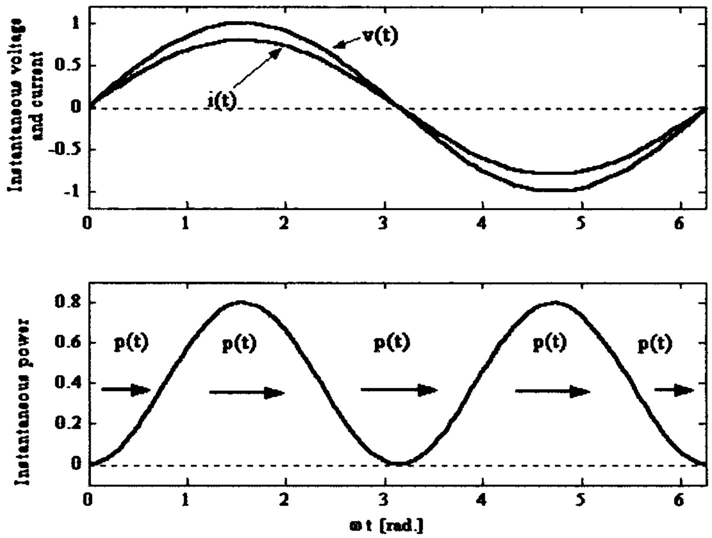
If the line is not naturally loaded, the component of electromagnetic field created by charging current and the component of electromagnetic field created by load current do not match and, due to their pulsating nature, at least the part of the energy has to be exchanged with source. To avoid this energy exchange between source and line, the line has to be compensated at intermediate points along the line with capacitors/inductance that are used as energy storages that exchange the energy with line locally on the points of its connection with line. Obviously, ideal compensation of the transmission line should be uniformly distributed, flexible dynamic energy storages at every point of the line.

2.3 Phase Shift

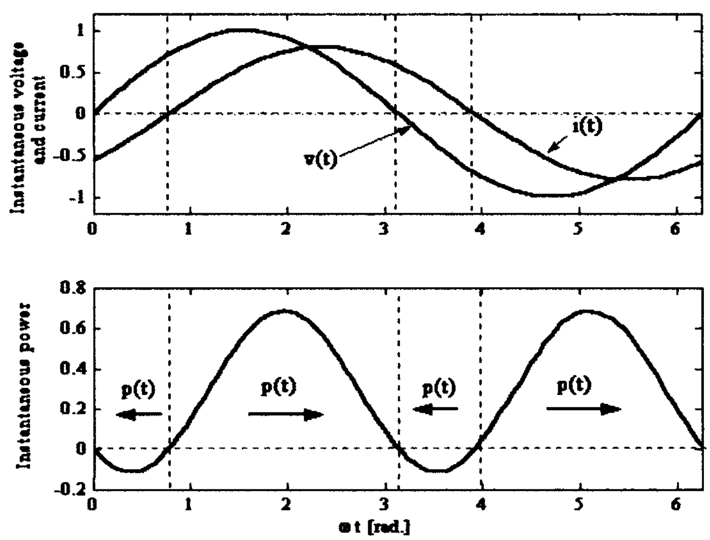
The cause of phase shift between voltage and current has been discussed above. When voltage and currents are in phase as shown in Fig.5 a) the instantaneous power (product of voltage and current) do not change sign as it can be seen on Fig.5.a) (lower trace). The physical meaning of the above mentioned fact is that power flow is only in one direction (from the source to the load). When the voltage and current through the line are phase shifted as shown on Fig.5 b) (upper trace) then instantaneous power change the sign four time over the one period of nominal frequency indicating the fact that the power flow changes direction four time over the one period, from the source to the load and from the load to the source. If the phase shift is 90° the net power flow is zero meaning that the quarter of the period power flows from the source to the load and next quarter period in opposite direction, from the load to the source. The net power transferred over the line (average power) is given by (2.19):

$$P = \frac{1}{T}\int_{0}^{T} p(t)dt \qquad (2.19)$$

Geometrically, the average power is the surface the instantaneous power curb *p(t)* forms with ωt axis. In case of the zero phase shift as in Fig.5.a) the surface is always above ωt axis (positive). In the case of phase shift between voltage and current the part of the surface the instantaneous power curb *p(t)* makes with ωt axis is above and part is bellow ωt axis.



(a)



(b)

Figure 5. a) When instantaneous voltage and current are in phase (upper trace) the instantaneous power flow is in one direction only (lower trace), b) when voltage and current are phase shifted (upper trace) the instantaneous power flow change direction four time over the one cycle (lower trace).

The part of the surface that is bellow ωt axis is considered to be negative and is subtracted from the part of the surface above ωt axis. It means that, for the same amplitude of the voltage and current, the net power transferred from the source to the load is lower for the phase shifted voltage and current. Because of the phase shift, for the constant voltage, to provide the same average power, the current has to be increased. Increased current means higher losses and higher inductive as well as resistive voltage drop.

The same principles apply to three phase circuit. The advantage of the three phase balanced system over the one phase system is in the fact that instantaneous power is not pulsating but constant physical entity.

2.4 Nature of Voltage Boost

Consider simple circuit shown in Fig.6. illustrating the bus voltage *v(t)* that is to be regulated and a compensator, that is represented as a voltage source $v_c(t)$ behind inductance L. The inductance L can be a transformer leakage reactance.

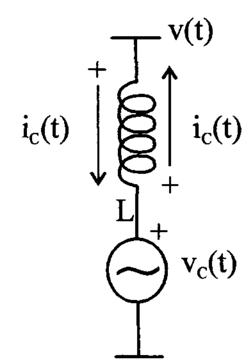


Figure 6 Voltage support of line voltage is provided with voltage source behind inductance L.

If the compensator voltage $v_c(t)$ is synchronized (in phase) with the bus voltage *v(t)* that is to be regulated, the net (average) active power flow between two of them is zero. As two voltages are in phase, the current $i_c(t)$ flowing between two of them is consequence of the voltage gradient only (difference in magnitude). If the both voltages are of the same magnitude there is no current flow between them. The change in the magnitude of the compensator voltage $v_c(t)$ change the current polarity. When the current polarity is changed, the polarity of the voltage drop across the inductance L is changed too, as shown in Fig.6, so the voltage drop across the inductance L can change from voltage boost to voltage drop and vice versa, depending on polarity of the current $i_c(t)$:

$$v(t) = L\frac{di_c(t)}{dt} + v_{c(t)} \tag{2.20}$$

One may be tempted to think that voltage boost (drop) depends upon inductance value L. The compensator current $i_c(t)$ can be calculated as:

$$i_c(t) = \frac{1}{L}\int(V_{\max} - V_{c\max})\cos\omega t dt = \frac{1}{\omega L}(V_{\max} - V_{c\max})\sin\omega t \tag{2.21}$$

as consequence:

$$L\frac{di_c(t)}{dt} = L\frac{d\left(\frac{1}{\omega L}(V_{\max} - V_{c\max})\sin\omega t\right)}{dt} = \left((V_{\max} - V_{c\max})\cos\omega t\right) \tag{2.22}$$

as it can be seen from (2.22), the voltage boost does not depend upon L if the source of compensating current i_c is voltage source behind inductance because the current through the inductance depends on inductance itself.

2.4.1 Phasor Approach

As the voltages and currents are sinusoidal it is helpful to use phasor notation.

$$v(t) = V_{\max}\cos\omega t = \mathrm{Re}\left\{V_{\max}e^{j\omega t}\right\} \tag{2.23}$$

$$v_C(t) = V_{C\max}\cos\omega t = \mathrm{Re}\left\{V_{C\max}e^{j\omega t}\right\} \tag{2.24}$$

$$\begin{aligned} i_c(t) &= \frac{1}{L}\int (V_{\max} - V_{c\max})\cos\omega t dt = \frac{1}{L}(V_{\max} - V_{c\max})\int \mathrm{Re}\left\{e^{j\omega t}\right\}dt = \\ &= \frac{1}{L}(V_{\max} - V_{c\max})\mathrm{Re}\left\{\frac{e^{j\omega t}}{j\omega}\right\} = \frac{1}{\omega L}(V_{\max} - V_{c\max})\sin(\omega t) = \\ &= \frac{1}{\omega L}(V_{\max} - V_{c\max})\cos(\omega t - 90^\circ) \end{aligned} \tag{2.25}$$

or, using phasor notation

$$\bar{I}_C = -\frac{j}{\omega L}(V_{\max} - V_{c\max}) \tag{2.26}$$

From (2.23) it can be seen that if $V_{max} > V_{Cmax}$ the current $i_c(t)$ lagging after the voltages $v_c(t)$ and $v(t)$ by 90^0 (inductive current) and if $V_{max} < V_{Cmax}$ the current $i_c(t)$ is leading the voltages $v_c(t)$ and $v(t)$ by 90^0 (capacitive current). The two cases are represented by Fig.7.a) and b).

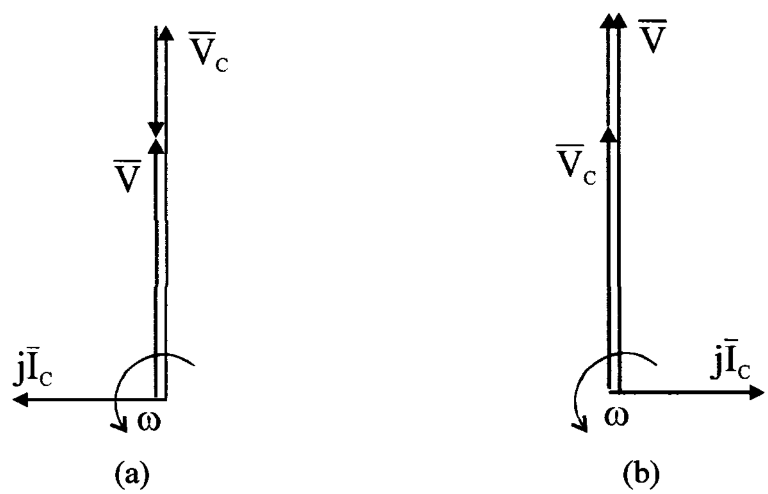


Figure 7 a) Vector diagram showing creation of voltage boost and b) voltage drop across inductance L.

2.5 Voltage Drop Analysis

2.5.1 Equivalent Circuit

Every power system, regardless of its topology, seen from the point of coupling with power delivery substation can be represented as Thévenin equivalent circuit as illustrated in Fig.8. The Thévenin voltage V_{Th} and impedance Z_{Th} can be readily obtained from short circuit current I_{SC}, short circuit capacity S_{SC} and X/R ratio of the supply system as:

$$S_{sc} = \overline{V}_{Th}\overline{I}^{*}_{SC} = \frac{\overline{V}^{2}_{Th}}{Z^{*}_{Th}} \tag{2.27}$$

$$\tan\phi_{sc} = \frac{X}{R}$$

where Z_{Th}=R+jX

These parameters are well known in engineering practice. The equivalent impedance Z_{Th} is usually inductive in meshed system, however it can be capacitive in case of long lines. The Thévenin impedance Z_{Th} can be considered as fictional, radial transmission line. The Thévenin voltage V_{Th} can be considered as sending end voltage V_S. V_R and V_L stand for receiving end and load voltage respectively while X_T is power delivery substation reactance and I_L load current.

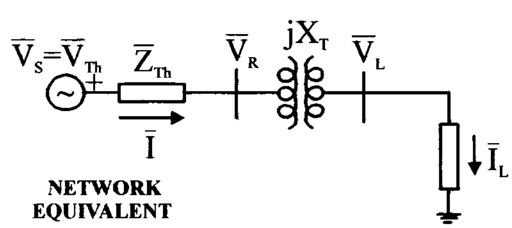


Figure 8 Power system seen from point of coupling with power delivery substation.

2.5.2 Non-Compensated Line

In order to get better insight in voltage drop in the lines and action to remedy it, it is useful to represent the line and the transformer as lumped inductance and consider simple, one line transmission system with one bulk power delivery transformer and load as shown in Fig.9 a). As it has been shown in the previous subsection it is very general case.

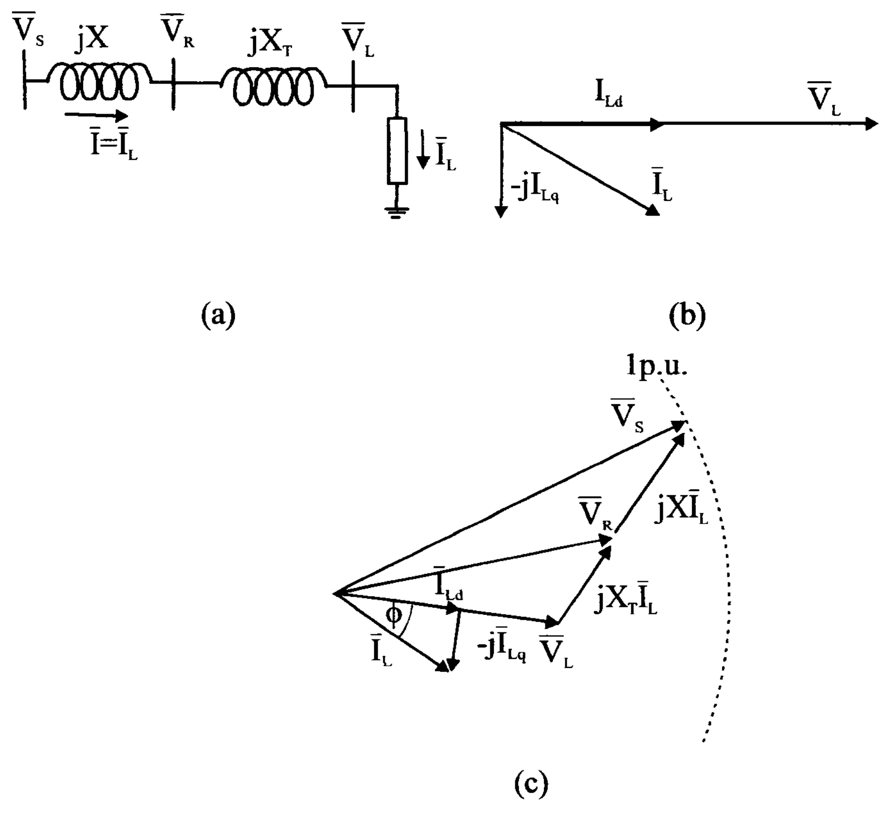


Figure 9 a) Single line diagram of radial transmission line (jX). Line is connected to infinite bus with voltage $\mathbf{V_S}$ = 1p.u. Transformer X_T connects line to load, b) Current phasor c) complete phasor diagram. Voltage drop is caused by the load current $\mathbf{I_L}$ across line reactance jX and supply transformer jX_T.

To highlight the essentials, following is assumed:

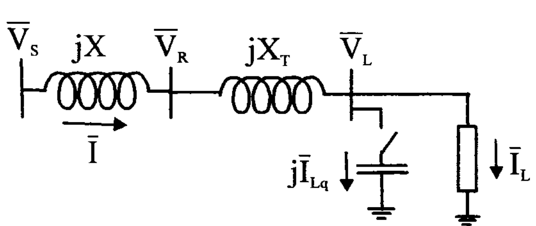
1. The line is connected to infinite bus.

2. The line is short and it can be represented by its series reactance jX.

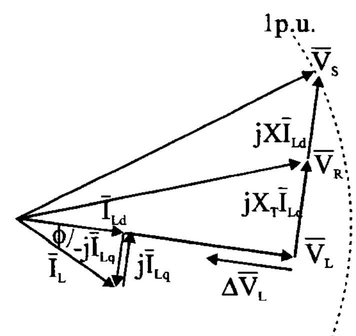
First we consider the non compensated line from Fig. 9. The current $\mathbf{I_L}$ drawn by the load is inductive and can be decomposed into two components, one that is in phase I_{Ld}, and another inductive $-jI_{Lq}$ in quadrature with load voltage $\mathbf{V_L}$ (Fig.9.b). The load current causes the voltage drop $j(X+X_T)(I_{Ld}-jI_{Lq})$ in the transformer and line reactance, decreasing receiving end voltage $\mathbf{V_R}$ and load voltage $\mathbf{V_L}$, as shown in Fig.9.c).

2.5.3 Power Factor Correction

The power factor correction indicate that all reactive requirements of the load are satisfied locally. The power factor of the load is mostly corrected with switched/fixed capacitors installed in parallel with load. After the power factor of the load is corrected locally by injecting capacitive current $+jI_{Lq}$, the inductive component of the load current $-jI_{Lq}$ and the voltage drop $\Delta\mathbf{V_L} = (X+X_T)I_{Lq}$ it causes are cancelled, as shown in Fig.10 b). The voltage drop caused by the I_{Ld} component of the load current still persists, preventing load voltage from being 1pu.



(a)



(b)

Figure 10 a) The reactive requirement of the load are supplied locally. b) After power factor correction, voltage drop is partially mitigated.

2.5.4 Voltage Regulation

The compensator is treated as reactive current source. To cancel voltage drop and to keep load voltage $\mathbf{V_L}$ at nominal value, the additional capacitive current has to be injected into the system. The capacitive current can be injected on distribution or on transmission level as illustrated in Fig. 11. If the voltage support is provided on distribution level (switch S_2 on and S_1 off) the injected capacitive current is perpendicular to distribution voltage $\mathbf{V_L}$. If voltage support provided on transmission level(switch S_2 off and S_1 on) the injected capacitive current is perpendicular to nodal transmission voltage $\mathbf{V_R}$. In both cases the line current **I** is vector sum of load current and compensation current.

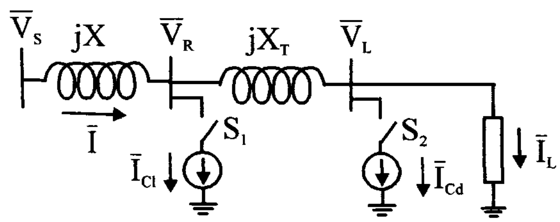


Figure 11 Single line diagram of radial transmission line (jX). Line is connected to infinite bus with voltage V_S = 1p.u. Transformer X_T connects line to load. Voltage support can be provided on transmission level (S_1 on, S_2 off) or on distribution level (S_1 off, S_2 on).

First we consider the non compensated line (Fig. 11 with switches S_1 and S_2 off). The power factor of the load is assumed to be corrected as shown previously. The current drawn by the load depends on load itself and the voltage $\mathbf{V_L}$. The current causes the voltage drop in the transformer and line reactance. It results in decrease in transmission voltage $\mathbf{V_R}$ and the load voltage $\mathbf{V_L}$. The load voltage $\mathbf{V_L}$ is in phase with current drown by the load. This is represented by the vector diagram as shown in Fig.12.

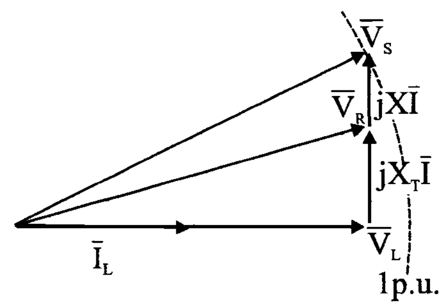


Figure 12 Voltage drop caused by the load current $\mathbf{I_L}$ across the line reactance jX and distribution transformer jX_T

Linearity of the circuit in Fig.11 makes it possible to apply superposition theorem in order to compare contributions of each compensator in voltage support. The circuit in Fig.11 can be represented as a sum of three circuits shown in Fig. 13. a), b) and c).

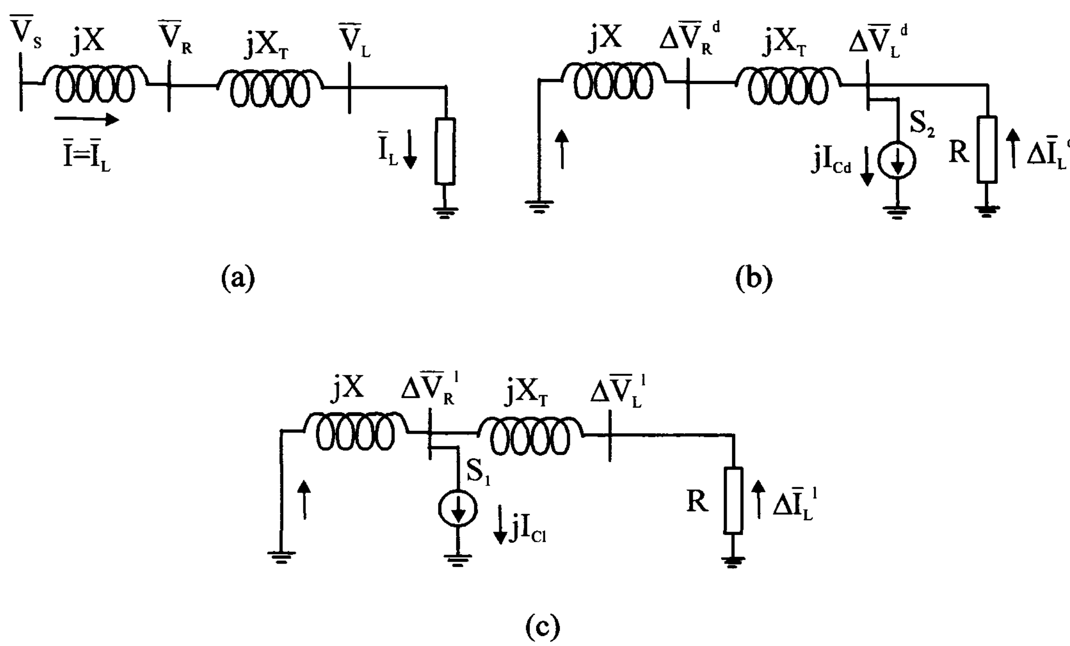


Figure 13 Circuit from Fig.11 decomposed according to the principle of superposition (a) without compensation, (b) voltage support provided on distribution side of power delivery substation, (c) voltage support provided on transmission side of power delivery substation

From Fig.13. b) it is possible to evaluate value of the voltage support due to reactive current injection $\mathbf{I_{Cd}}$ on the distribution side of substation:

$$\Delta\bar{I}_L^d = \frac{jI_{Cd}j(X+X_T)}{R+j(X+X_T)} \tag{2.28}$$

$$\Delta\bar{V}_L^d = -R\Delta\bar{I}_L^d = \frac{RI_{Cd}(X+X_T)}{R+j(X+X_T)} \tag{2.29}$$

$\Delta V_L{}^d$ is voltage boost of the load voltage $\mathbf{V_L}$ due to current $\mathbf{I_{Cd}}$ injected by the compensator on the load side. The plus sign indicate that it is voltage drop, but due to the fact that the voltage drop given by (2.28) is in opposite direction according to the voltage drop caused by the load current $\mathbf{I_L}$ in transformer and line reactance $\mathbf{I}j(X+X_T)$ in Fig.11, it is actually the voltage boost.

Influence of injected compensating current $\mathbf{I_{Cd}}$ on distribution side of the substation on transmission voltage is given by:

$$\Delta\bar{V}_R^d = \frac{RjI_{Cd}jX}{R+j(X+X_T)} = -\frac{RI_{Cd}X}{R+j(X+X_T)} \tag{2.30}$$

Equation (2.30) give voltage boost on the transmission side of the substation. (Minus sign depicts the fact that voltage drop is negative (negative voltage drop is boost), and the part of the compensator current flowing through the line impedance jX has the same direction as total current flowing through the line - the current I in Fig.11.

If the compensation current is injected on transmission side of substation as shown in Fig.13 c)

$$\Delta \bar{I}_L^l = \frac{jI_{Cl}jX}{R + j(X + X_T)} \tag{2.31}$$

$$\Delta \bar{V}_L^l = -R\Delta \bar{I}_L^l = \frac{RI_{Cl}X}{R + j(X + X_T)} \tag{2.32}$$

$$\Delta \bar{V}_R^l = \frac{(R + jX_T)jI_{Cl}jX}{R + j(X + X_T)} = -\frac{XI_{Cl}(R + jX_T)}{R + j(X + X_T)} \tag{2.33}$$

if both injected currents are equal in magnitude $|I_{Cd}| = |I_{Cl}|$ then from (2.29) and (2.30)

$$\frac{\Delta \bar{V}_L^d}{\Delta \bar{V}_L^l} = \frac{(X + X_T)}{X} \tag{2.34}$$

From (2.34) it can be seen that both placement of compensators provide voltage boost on distribution side of the transformer X_T, but the voltage boost is more pronounced when compensator sat on distribution side due to transformer reactance X_T.

The influence of two compensating schemes on transmission side voltage $\mathbf{V_T}$ can be compared from (2.30) and (2.33). Again, for same magnitude of the compensating currents

$$\frac{\Delta \bar{V}_R^d}{\Delta \bar{V}_R^l} = \frac{R}{R + jX_T} \tag{2.35}$$

From (2.35) it can be seen that in both cases transmission voltage $\mathbf{V_R}$ is supported, but the support of transmission voltage is more efficient in case of compensator on high voltage side of the transformer X_T.

2.6 Conclusion

In this chapter the mechanisms of power transfer, voltage drop and voltage boost has been discussed. It has been shown that the nature of the inductive voltage drop and voltage boost is in the phenomenology of electromagnetic field and Faraday-Lenz law. Moreover, it has been shown that voltage drop across the inductance can be changed into voltage boost if the polarity of the current through inductance is changed.

Finally, a simple power system consisting of the infinite bus, radial transmission line, power delivery transformer and load has been analyzed using principle of superposition. The influence of the voltage support provided on transmission and distribution level has been analyzed. It has been shown that, when voltage support is provided on distribution level the transmission voltage is supported too and vice versa.